Electromotive Force and Circuits

Lecture Outline
1. Electromotive Force
2. Kirchoff's Rules
2. Resistors in Series
3. Resistors in Parallel
4. Multi loop circuits
5. Electrical Meters
6. RC Circuits

Electromotive Force (EMF)

A direct current circuit is defined as a type of circuit in which charge flows smoothly, connected to a potential source, with simple circuit elements connected in series or parallel.

EMF:

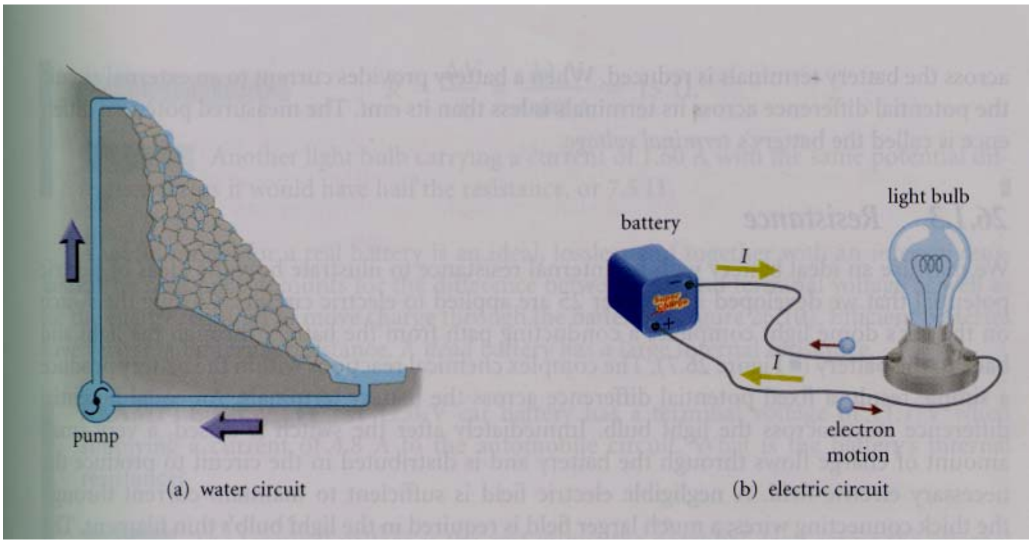
A device which increase the potential energy of charge circulating within an electric circuit is termed a source of **emf**, symbolized as ε and sometimes referred to as "electromotive force".

EMF has the same unit as the potential, which in SI unit is the volt.

As an example, a battery is a source of emf, converting chemical potential energy into electrical potential energy. The potential across the terminals of a battery is not in general equal to the battery emf, due to the non-zero internal resistance within a battery. Terminal voltage for a battery is given as:

$$\Delta V = \varepsilon - I \times r$$

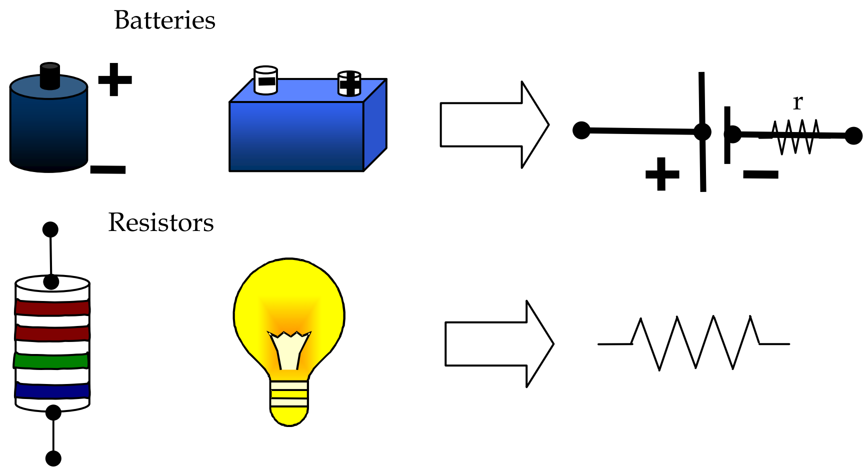
Batteries and Electromotive force



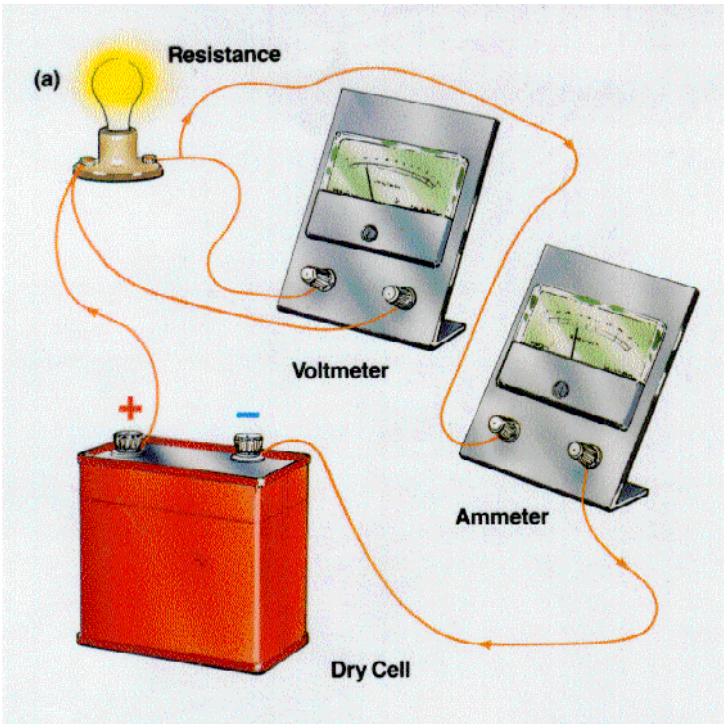
(a) A water circuit consists of a pump and a continuous stream of water that flows downhill through a pile of rocks. (b) In an electric circuit, the battery raises the electric potential of charge in the same way that pump raises the gravitational potential of water. Here the arrow shows the direction of electric current I. The electrons move in the opposite direction.
We can make analogy between the flow of electrons in a wire and fluid flow.
Pump ←→ battery
Pile of rocks ←→ light bulb
Within a battery, a chemical reaction occurs that transfers electrons from one terminal (leaving it positively charged) to another terminal (leaving it negatively charged).

The ***electromotive force*** of a battery or other electric power source is the value of the potential difference it maintains between its terminals in the absence of current. In a typical car battery, the chemical reaction maintains the potential difference at a maximum of 12 volts between the positive and negative terminals, so the emf is 12 V. In a typical flashlight battery the emf is 1.5 V. The batteries consist of an internal small resistance r.

Circuit symbols



Kirchoff's Rules

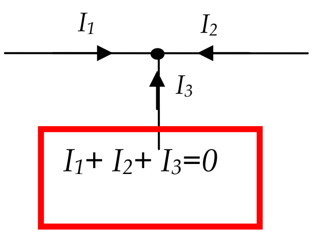


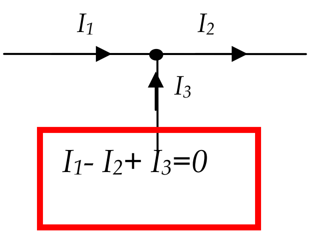
In a circuit, charges move from one place to another carrying energy. These charges can be thought of as buckets that carry energy around a circuit. The battery fills the buckets. The buckets are emptied at various places around the circuit, but the buckets themselves never disappear. They return to the battery to be refilled. These basic ideas are summarized in Kirchoff's Rules and are applicable to even the most complicated circuits.

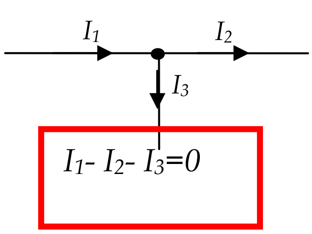
The Junction Theorem:

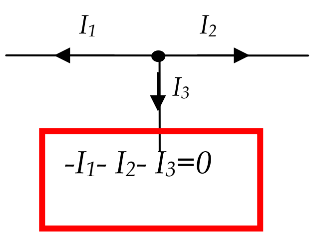
"The current into any junction is exactly equal to the current out of the junction."

This theorem is explained by the Law of Conservation of Charge.



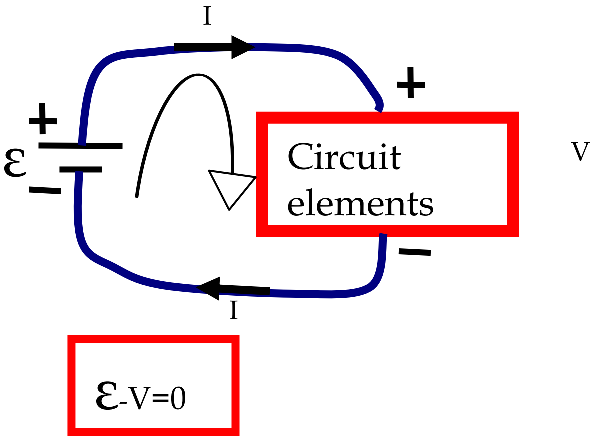




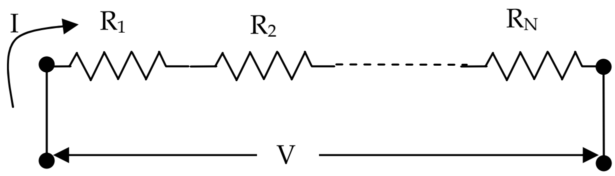


The Loop Theorem:

"The sum of all the voltage drops around any loop in a circuit must be zero." This theorem is explained by The Law of Conservation of Energy and the fact that the electric force is conservative.



Resistors in Series



The loop theorem requires:

$$V - V_1 - V_2 - \cdots - V_N = 0 \Rightarrow V = V_1 + V_2 + \cdots + V_N$$

Ohm's Law says: $V = IR,\ V_1 = I_1R_1,\ V_2 = I_2R_2,\ \cdots V_N = I_NR_N$

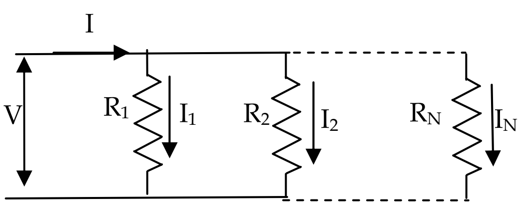
$$\Rightarrow IR = I_1R_1 + I_2R_2 + \cdots + I_NR_N$$

The junction theorem means that: $I = I_1 = I_2 = \cdots = I_N \Rightarrow IR = IR_1 + IR_2 + \cdots + IR_N$

$$\Rightarrow R = R_1 + R_2 + \cdots + R_N$$

$$\boxed{R_s = \sum R_i \quad \text{Resistors in Series}}$$

Resistors in Parallel



The junction theorem means that:

$$I = I_1 + I_2 + \cdots + I_N$$

Ohm's Law says:

$$I = \frac{V}{R},\ I_1 = \frac{V_1}{R_1},\ I_2 = \frac{V_2}{R_2},\ \cdots I_N = \frac{V_N}{R_N} \Rightarrow \frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_N}{R_N}$$

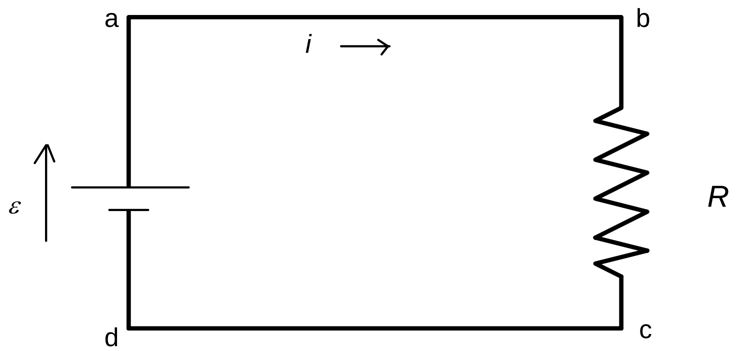
The loop theorem requires:

$$V - V_1 = 0,\ V - V_2 = 0,\ \cdots V - V_N = 0 \Rightarrow V = V_1 = V_2 = \cdots = V_N$$

So, $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_N} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$

$$\boxed{\frac{1}{R_p} = \sum \frac{1}{R_i} \quad \text{Resistors in Parallel}}$$

Example:
For the following circuit, what is the potential drop a) from point *a* to *b*, b) from point *a* to *c*, c) from point *b* to *a* and d) from point *c* to *a*? Assume to wires to have negligible resistance.



a) From point *a* to *b* we have no change in the potential since there is no resistors or emf sources present.

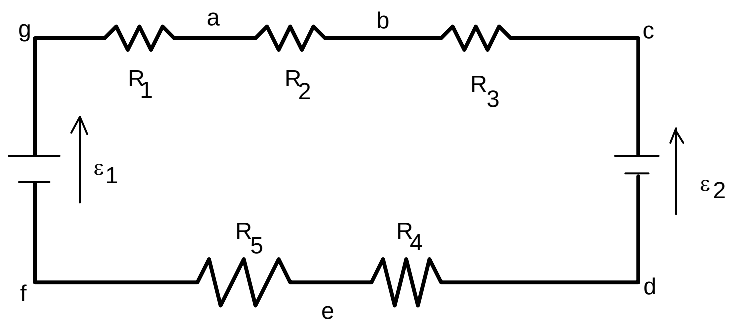
b) From point *a* to *c* we have a resistor, so we get a potential drop of $V = iR$.

c) From point *b* to *a* we first encounter a resistor, which causes a potential drop of $V = iR$, then the emf source, which causes a potential gain of $V = iR$ (we know that $\varepsilon = R$ since the potential drop for the whole loop must be zero). Thus we get that

$$V = -iR + iR = 0.$$

d) From point *c* to *a*, we have an emf source, so we get a potential gain of $V = iR$.

Example:
Find the current in the following circuit.



In this circuit we have two sources of emf and three resistors. We cannot predict the direction of the current unless we know which emf is greater, but we do not have to know the direction of the current before solving the problem. We can assume any direction and solve the problem with that assumption. If the assumption is incorrect, we shall get a negative number for the current, indicating that its direction is opposite to that assumed.

Let us assume *i* to be clockwise. The potential drops and increases as we traverse the circuit in the assumed direction of the current, starting at point *a*, are (where a negative sign indicates a potential drop)

$V_{ab} = -iR_2$ $\quad$ $V_{bc} = -iR_3$

$V_{cd} = -\varepsilon_2$ $\quad$ $V_{de} = -iR_4$

$V_{ef} = -iR_5$ $\quad$ $V_{fg} = \varepsilon_1$

$V_{ga} = -iR_1.$

We encounter a potential drop traversing one of the emfs and an increase traversing the other. The loop theorem gives

$$\varepsilon_1 - \varepsilon_2 = i(R_1 + R_2 + R_3 + R_4 + R_5)$$

which, when solved for i, yields

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2 + R_3 + R_4 + R_5}$$

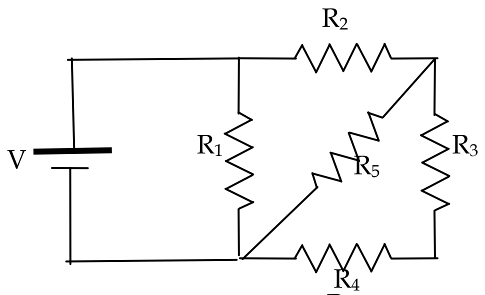
Notice that if ε_2 is greater that ε_1, we get a negative number for the current i, indicating that we have chosen the wrong direction for i. For ε_2 greater than ε_1 the current is in the counterclockwise direction. also notice that while the stronger emf source is **discharging**, the weaker emf source is **charging**.

Multi loop Circuits

1) Circuit elements in series have the same current, but divide up a common voltage.
2) Circuit elements in parallel have the same voltage, but divide up a common current.

Many resistor circuits are just combinations of series and parallel. They can be studied using the series and parallel rules. This is called "circuit reduction." Other circuits are not combinations of series and parallel. These circuits have to be examined with Kirchoff's Rules.

Example For the circuit shown find (a)the equivalent resistance, (b)the current provided by the battery and (c)the current through and voltage across each resistor (V=90.0V, R_1=3.00kΩ, R_2=4.00kΩ, R_3=1.00kΩ, R_4=2.00kΩ and R_5=6.00kΩ).

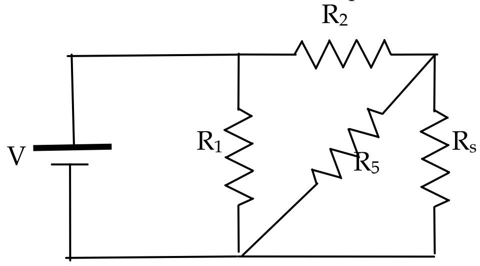


(a)Use the idea of circuit reduction.

R_3 and R_4 are in series.

They can be replaced with an equivalent resistor,

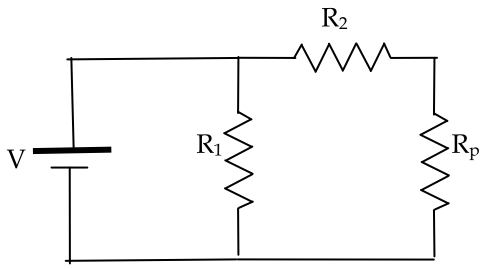
$R_s = R_3 + R_4 = 3.00\text{k}\Omega$.



R_s and R_5 are in parallel.

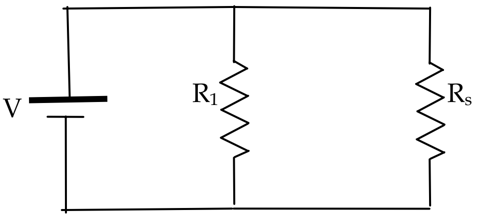
They can be replaced with an equivalent resistor,

$$\frac{1}{R_p} = \frac{1}{R_s} + \frac{1}{R_5} \Rightarrow R_p = 2.00\text{k}\Omega.$$



R_p and R_2 are in series.

They can be replaced with an equivalent resistor,
$R_s = R_p + R_2 = 6.00\,k\Omega$.
R_S and R_1 are in parallel. The equivalent resistor is,
$\frac{1}{R} = \frac{1}{R_s} + \frac{1}{R_1} \Rightarrow R = 2.00\,k\Omega$.

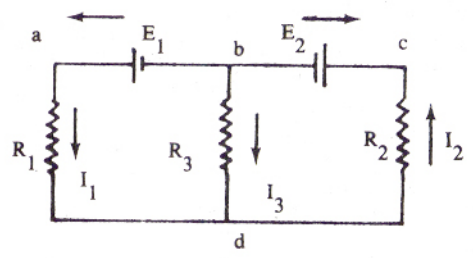


(b)Use Ohm's Rule
$V = IR \Rightarrow I = \frac{V}{R} = \frac{90.0V}{2.00\,k\Omega} = \underline{\underline{45.0\,mA}}$

(c)Follow the current and voltage drops around the circuit using Kirchoff's Rules and Ohm's Rule.
In summary,

V(V)	I(mA)	R(kΩ)
90.0	30.0	3.00
60.0	15.0	4.00
10.0	10.0	1.00
20.0	10.0	2.00
30.0	5.00	6.00

Example: For the given circuit find the current flows and potential drop on each resistor.



E_1=2.0 V $\quad$ E_2=1.0 V
R_1=10 Ω $\quad$ R_2=R_3=20 Ω

Solution

From the 1st rule: $\quad \sum I_{in} = \sum I_{out}$

at junction d, $\quad I_1 + I_3 = I_2$

or $\quad I_3 = I_2 - I_1 \quad (1)$

From the 2nd rule: $\quad \sum E = \sum RI$

Loop adb: $\quad E_1 = I_1R_1 - I_3R_3$

Then $\quad 2 = 10I_1 - 20I_3 \quad (2)$

Loop bcd: $\quad E_2 = -I_2R_2 - I_3R_3$

Then $\quad 1 = -20I_2 - 20I_3 \quad (3)$

(2)-(3) $\quad 1 = 10I_1 + 20I_2 \quad (4)$

putting (1) into (2): $\quad 2 = 10I_1 - 20(I_2 - I_1)$

$2 = 10I_1 - 20I_2 + 20I_1$

$2 = 30I_1 - 20I_2 \quad (5)$

$3 = 40I_1$

(4)+(5) $\quad I_1 = \frac{3}{40}$

$$1 = 10\left(\frac{3}{40}\right) + 20I_2$$

substituting I_1 into (4):

$$1 = \frac{3}{4} + 20I_2$$

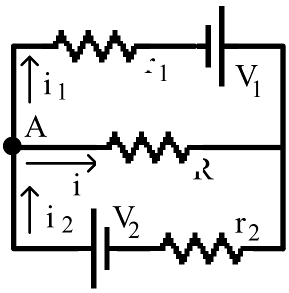
$$20I_2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$I_2 = \frac{1}{80}$$

Therefore, (1) $\quad I_3 = \frac{1}{80} - \frac{3}{40}$

$$I_3 = -\frac{5}{80} = -\frac{1}{16}\ \text{A}$$

The sign for I_3 is negative. That means our assumed direction is wrong. I_3 should flow from d to b.



Example: A 6.00V battery (V_1) with a 2.00Ω internal resistance and a 3.00V battery with a 3.00Ω internal resistance is connected to a 6.00Ω resistor as shown. Find the terminal voltage for each battery and the current through the 6.00Ω resistor.

This circuit is not a combination of series and parallel resistor, so we must go back to the more basic ideas of Kirchoff's Rules to solve the problem. Applying the junction theorem at point A $\Rightarrow i_2 = i + i_1$. The loop theorem around the lower loop $\Rightarrow V_2 - iR - i_2 r_2 = 0$ and around the upper loop $\Rightarrow V_1 + iR - i_1 r_1 = 0$.

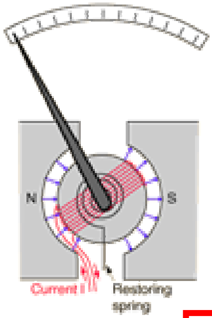
This gives three equations for the three unknown currents. Solving the loop theorem equations for i_1 and i_2,

$i_2 = \dfrac{V_2 - iR}{r_2}$ and $i_1 = \dfrac{V_1 + iR}{r_1}$. Substituting into the junction theorem equation, $\dfrac{V_2 - iR}{r_2} = i + \dfrac{V_1 + iR}{r_1}$.

Solving for i, $i = \dfrac{V_2 - V_1\dfrac{r_2}{r_1}}{R + r_2 + R\dfrac{r_2}{r_1}} = \underline{\underline{-0.333\text{A}}}$ the minus sign means that we chose the direction of this current wrong. Substituting back for i_1 and i_2, $i_1 = \underline{\underline{2.00\text{A}}}$ and $i_2 = \underline{\underline{1.67\text{A}}}$.

The terminal voltage is the actual potential difference across the terminals of the battery, $V_{t1} = V_1 - i_1 r_1 = \underline{\underline{2.00\text{V}}}$ and $V_{t2} = V_2 - i_2 r_2 = \underline{\underline{-2.00\text{V}}}$

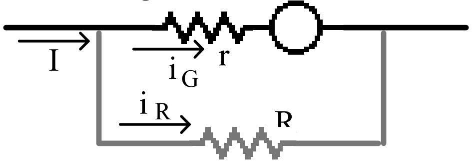
Electrical Measuring instruments



Galvanometer | Voltmeter | Ampermeter | Ohmmeter

The basic constituent part of a voltmeter, ammeter, or ohmmeter is a galvanometer. Mechanical galvanometers are made from a coil of wire and a magnet. Modern galvanometers are IC chips. They all function in the same basic way. They respond linearly to small currents. The different types of meters are constructed from a galvanometer and properly placed resistors.
The schematic symbols for different types of meters are shown at the right.

Example: A galvanometer of internal resistance 0.120Ω reads full scale when 15.0μA passes through it. Design an ammeter to read up to 1.00A with this galvanometer.

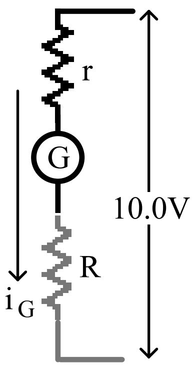


If the entire 1.00A goes through the galvanometer it will blow up. We must provide an alternate path for the current. We can control the fraction of the current that goes through the galvanometer with a resistor called a "shunt."

Using the junction theorem, $I = i_G + i_R$. The loop theorem requires $i_G r - i_R R = 0$. Solving for i_R and substituting into the junction theorem equation ,

$$i_R = \frac{r}{R} i_G \Rightarrow I = i_G\left(1 + \frac{r}{R}\right) \Rightarrow R = \frac{r}{\left(\frac{I}{i_G}\right) - 1} = \underline{\underline{1.80\mu\Omega}}.$$

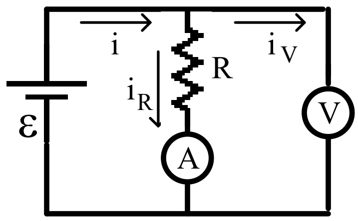
Example 4: Use an identical galvanometer to build a voltmeter to measure up to 10.0V.



If the entire 10.0V is across the galvanometer the current will be huge and it will cook. We must cut down the current that goes through the galvanometer with a resistor in series. Using the loop theorem, $V = i_G r + i_G R$.

Solving for R, $R = \frac{V}{i_G} - r = \underline{\underline{6.67\text{x}10^5\Omega}}$.

Example: The ammeter from example 3 and the voltmeter from example 4 are used to measure the resistance in the circuit shown. Find the difference between the ratio of the meter readings and the true resistance.



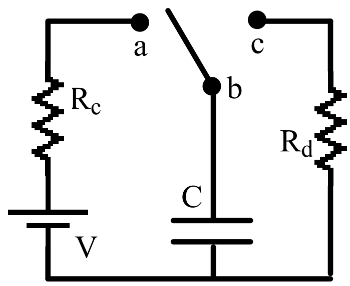
Using the loop theorem, $V = i_R(R + R_A)$ where V is the voltmeter reading and R_A is the resistance of the ammeter which can be found from the parallel rule,

$$\frac{1}{R_A} = \frac{1}{r} + \frac{1}{R_{shunt}} \Rightarrow R_A = \frac{rR_{shunt}}{r + R_{shunt}} \approx R_{shunt} = 1.80\mu\Omega.$$

Using $R' \equiv \frac{V}{i_R}$ as the ratio of the meter readings the equation from the loop theorem can be written, $i_R R' = i_R(R + R_A) \Rightarrow R' = R + R_A = R + R_{shunt}$. The difference between the meter ratio and the actual resistance is, $R' - R = R_{shunt} = \underline{\underline{1.80\mu\Omega}}$.

RC Circuits

Kirchoff's Rules are very widely applicable. This is not surprising considering they come from the Laws of Conservation of Energy and Conservation of Charge. They can be used to analyze circuits with both resistors and capacitors.



A typical RC circuit is shown at the right. When the switch connects a and b current flows and the battery begins to charge the capacitor. When the capacitor is charged current can no longer flow. The question is, how long does this take?

At some intermediate time the current in the circuit is i and the charge on the capacitor is q. Applying the loop theorem, $V - iR_c - \frac{q}{C} = 0$. From the definition of current, the current must equal the rate at which the capacitor charges, $i = \frac{dq}{dt}$. The equation from the loop theorem becomes, $V - \frac{dq}{dt}R_c - \frac{q}{C} = 0$. This equation can be solved for q(t) by solving for $\frac{dq}{dt}$ and integrating,

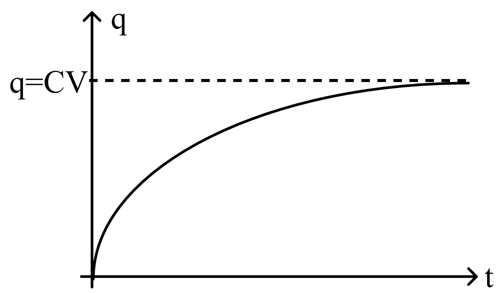
$$\frac{dq}{dt} = \frac{CV - q}{R_c C} \Rightarrow \frac{dq}{CV - q} = \frac{dt}{R_c C} \Rightarrow \int_0^q \frac{dq}{CV - q} = \int_0^t \frac{dt}{R_c C} \Rightarrow -\int_{CV}^{CV-q} \frac{du}{u} = \frac{t}{R_c C} \Rightarrow \ln\left(\frac{CV - q}{CV}\right) = -\frac{t}{R_c C}$$

Solving for q,

$$\boxed{q = CV\left(1 - e^{-t/RC}\right) \quad \text{Charging RC Circuit}}$$

The graph of the charge on the capacitor as a function of time is shown.

Note:
1)The charge is initially zero.
2)The charge grows exponentially.
3)The maximum charge occurs when the capacitor voltage matches the battery voltage.



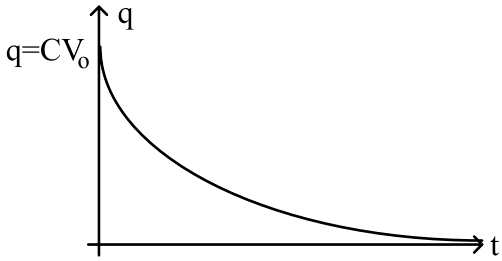
When the switch connects b and c the capacitor discharges. The loop theorem requires, $-iR_d + \frac{q}{C} = 0$. The current must equal the rate at which the capacitor discharges, $i = -\frac{dq}{dt}$.

The equation from the loop theorem becomes, $\frac{dq}{dt} = -\frac{q}{R_d C}$.

$$\text{Integrating, } \frac{dq}{q} = -\frac{dt}{R_d C} \Rightarrow \int_{CV_o}^{q} \frac{dq}{q} = -\int_0^t \frac{dt}{R_d C} \Rightarrow \ln\left(\frac{q}{CV_o}\right) = -\frac{t}{R_d C} \Rightarrow q = CV_o e^{-t/R_d C}$$

$$\boxed{q = CV_o e^{-t/RC} \quad \text{Discharging RC Circuit}}$$

The graph of the charge on the capacitor as a function of time is shown.



Note:
1)The charge is initially the capacitance times the initial voltage on the capacitor.
2)The charge dies exponentially.

Example 6: A 5.00μF capacitor is charged to 10.0V. A 10.0cm piece of 2.00mm diameter copper wire is used to short it out. Find the time it takes for the capacitor's voltage to drop to 10.0mV.

For the discharge of a capacitor, $q = CV_o e^{-t/RC} \Rightarrow \frac{q}{C} = V_o e^{-t/RC} \Rightarrow V = V_o e^{-t/RC}$.

Solving for the time, $\frac{V}{V_o} = e^{-t/RC} \Rightarrow \ln\left(\frac{V}{V_o}\right) = -\frac{t}{RC} \Rightarrow t = -RC\ln\left(\frac{V}{V_o}\right)$.

The resistance can be found from its definition,

$$R = \rho\frac{\ell}{A} = (1.7\text{x}10^{-8})\frac{0.100}{\pi(0.00100)^2} = 5.41\text{x}10^{-4}\,\Omega.$$

Putting the numbers in,

$$t = -RC\ln\left(\frac{V}{V_o}\right) = (5.41\text{x}10^{-4})(5.00\text{x}10^{-6})\ln\left(\frac{0.0100}{10.0}\right) = 1.9\text{x}10^{-8}\text{s} = \underline{\underline{19\text{ns}}}.$$

Example: A 6 V battery of negligible internal resistance is used to charge a 2 μF capacitor through a 100 Ω resistor. Find the initial current, the final charge, and the time required to obtain 90 percent of the final charge.

The initial current is $i_0 = \frac{\varepsilon}{R} = \frac{(6\text{ V})}{(100\ \Omega)} = 0.06\text{ A}$

The final charge is $q_f = \varepsilon C = (6\text{ V})(2\ \mu\text{F}) = 12\ \mu\text{C}$.

The time constant for this circuit is $RC = (100\ \Omega)(2\ \mu\text{F}) = 200\ \mu\text{sec}$. Setting $q = 0.9\ \varepsilon C$
Complete solution!

Summary

Kirchoff's Rules:

The Junction Theorem: "The current into any junction is exactly equal to the current out of the junction."

The Loop Theorem: "The sum of all the voltage drops around any loop in a circuit must be zero."

Resistors in Series $R_s = \sum R_i$

Resistors in Parallel $\frac{1}{R_p} = \sum \frac{1}{R_i}$

Charging RC Circuit $q = CV\left(1 - e^{-t/RC}\right)$

Discharging RC Circuit $q = CV_o e^{-t/RC}$