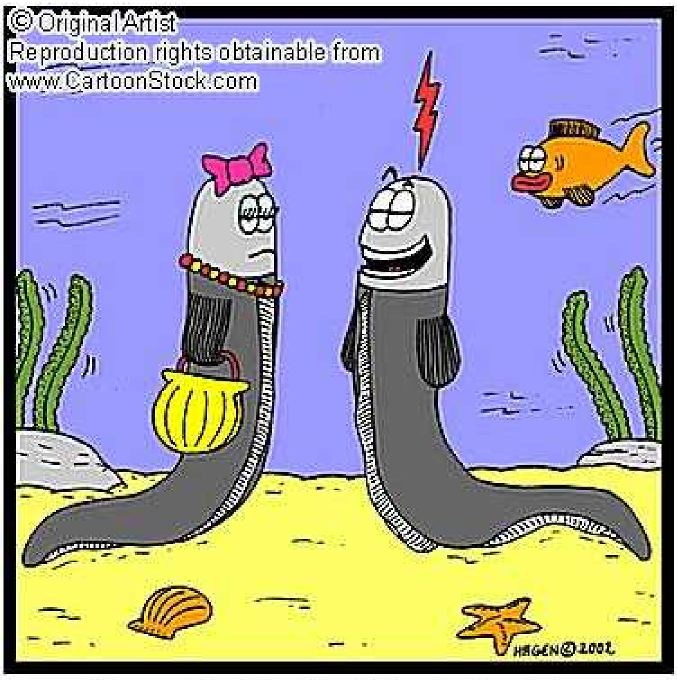
Chapter 2: Electric Field

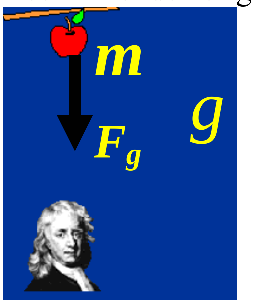


Can you feel the electricity in the air?...

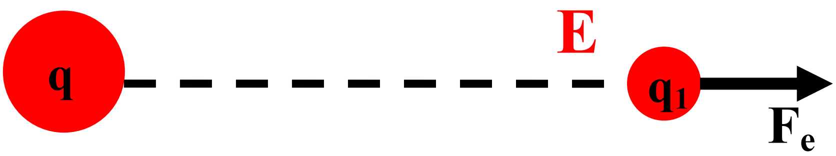
Lecture Outline
1. The Definition of Electric Field
2. Electric Field Lines
3. The Electric Field Due to Point Charges
4. The Electric Field Due to Continuous Charge Distributions
5. The Force on Charges in Electric Fields

Electric field

A charge can experience an electrostatic force due to the presence of other charges. The idea of a field is required in order to explain the "action at a distance."
Recall the idea of gravitational field.



In this view, Earth creates a force on the mass m. This is "insane." Earth isn't even touching the mass. So we introduce the idea of a gravitational field. Now we take the view that the field due to Earth, g, is exerting the gravitational force on the mass. Since the force is $\vec{F}_g = m\vec{g}$, the gravitational field is defined as, $\vec{g} \equiv \frac{\vec{F}_g}{m}$. Let's take the same approach with the electric force.



Instead of thinking of q exerting the force on q_1, we think of q creating a field and the field exerting the force on q_1.
Mathematically, we can write

$$\boxed{\vec{F}_e = q_1\vec{E} \Rightarrow \vec{E} = \frac{\vec{F}_e}{q_1}}$$

which is the definition of the electric field. The electric field is a vector, and its direction is the same as the direction of the force $\vec{F}$ on a positive test charge. It has units Newton per coulomb (N/C)
Note:
-The surrounding charges (not q) is the ones that create an electric field.
-Similarly between electric field and gravitational field

The electric field due to a point charge

$$\boxed{\vec{E} = \frac{kq}{r^2}\hat{r}}$$

Definition of the electric field

The electric field due to N charged particles

$$\boxed{\vec{E} = \sum_{i=1}^{n} \frac{kq_i}{r_{0i}^2}\hat{r}_{0i}}$$

Example: A point charge Q_1 = + 10.0 C is at the origin, and a second charge Q_2 = -5.0 C is placed on the y-axis at y = 1.0 m. What is the total electric field at the point P with coordinates x = 2.0 m, y = 1.0 m ?

We use Coulomb laws to find the field due to each charge, and then we find the vector sum.

First, we found the field $\vec{E}_1$ due to Q_1

$$\vec{E}_1 = k\frac{Q_1}{r_1^2}\hat{r}_1 = (9.0\times10^9\frac{N.m^2}{C^2})\frac{(10.0C)}{[(2.0m)^2+(1.0m)^2]}\frac{(2\hat{i}+\hat{j})}{\sqrt{5}} = [1.6\hat{i}+0.8\hat{j}]\times10^{10}N/C$$

The field at P due to the negative charge Q_2 is in the –x direction:

$$\vec{E}_2 = k\frac{Q_2}{r_2^2}\hat{r}_2 = (9.0\times10^9\frac{N.m^2}{C^2})\frac{(-5.0C)}{(2.0m)^2}\hat{i} = \hat{i}(-1.1\times10^{10}N/C)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = [1.6\hat{i}+0.8\hat{j}-1.1\hat{i}]\times10^{10}N/C$$

$$= [0.5\hat{i}+0.8\hat{j}]\times10^{10}N/C$$

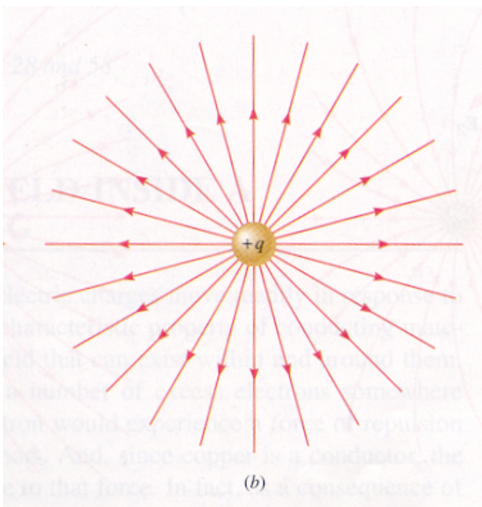
or $\vec{E}$ =(0.94x10^{10})N/C, in direction $0.5\hat{i}+0.8\hat{j}$

Electric field lines

We need a more descriptive image of the field. The most useful idea is "Electric Field Lines." Field Lines or Lines of Force are used to visualize the field. The rules for drawing them are:

1. The tangent to the field line points in the direction of the force on a positive test charge.
2. The density of lines is proportional to the strength of the field (bigger charges make more lines).

One way to visualize the electric field is drawing connecting lines of vector (electric field) and these connecting lines are called ***electric field lines***.



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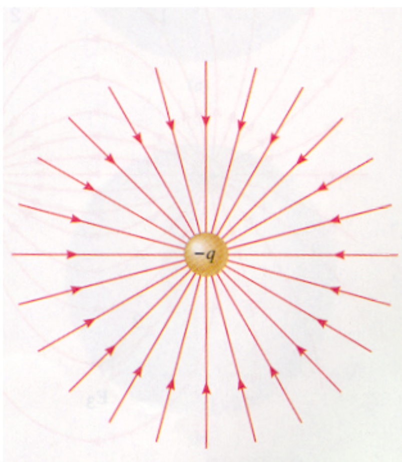
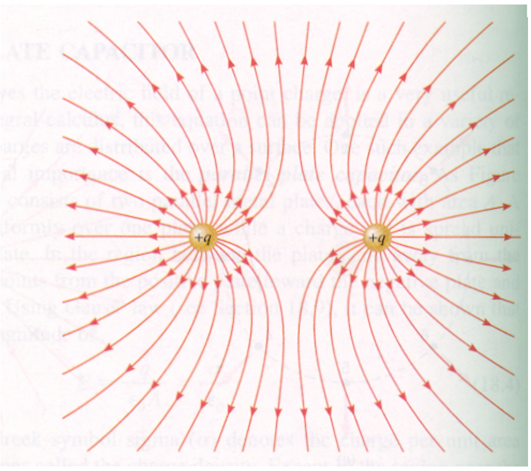
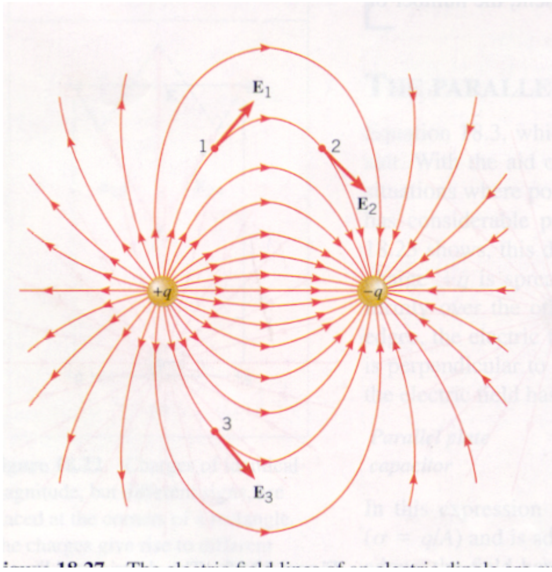






Calculation of Electric field due to point charges

In this section we will demonstrate calculation of electric field on illustrative examples.

Example 1

A proton and an electron form two corners of an equilateral triangle with sides of length 3×10^{-6} m. What is the magnitude of their net electric field at the third corner?

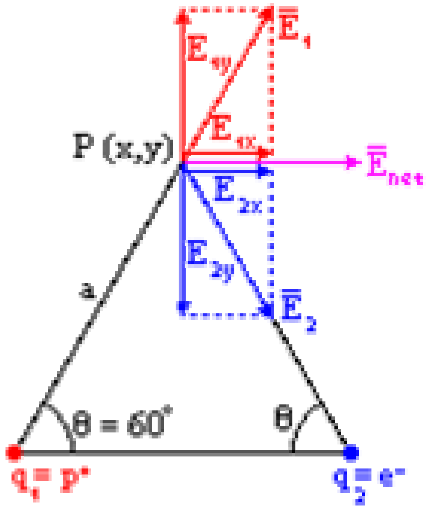
Answer:

For this problem we draw as a diagram of the charges and find **E** at point P(x,y).

Known:

$$\vec{E} = \sum_{i=1}^{n} \frac{kq_i}{r_{0i}^2}\hat{r}_{0i}$$

$q_1 = p^+ = 1.6 \times 10^{-19}$ C;
$q_2 = e^- = -1.6 \times 10^{-19}$ C;
$r = a = 3 \times 10^{-6}$ m



Red line: electric field vector of proton
Blue line: electric field vector of electron.
Since electric field is a vector quantity, we can write

$$\vec{E} = \vec{E}_e + \vec{E}_p$$

$$\vec{E}_e = E_e \cos 60^0 \,\hat{x} - E_e \sin 60^0 \,\hat{y}$$

$$\vec{E}_p = E_p \cos 60^0 \,\hat{x} + E_p \sin 60^0 \,\hat{y}$$

and magnitude of $E_e = E_p = k\dfrac{q}{a^2}; \quad q = 1.6\times10^{-19} C$

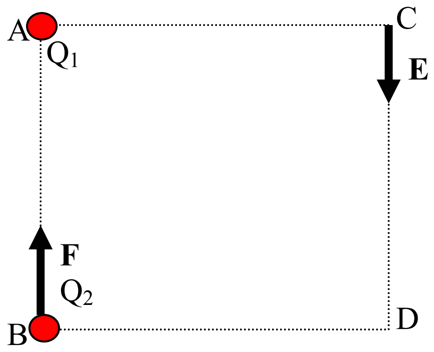
$$\vec{E} = 4.8\times10^{-4}\,\hat{x}\,(N/C)$$

Example 2:

Two charges are at the corners of a square of side a=1 cm. The net electric field at C due to the charges is along –y direction and force acting on the charge Q_2 due to Q_1 is along +y direction. If the magnitude of force is $\sqrt{2}$ N, determine:

a) magnitude of charges Q_1 and Q_2,
b) Magnitude of electric field **E.**

c) What charge must be placed at D so that the electric field at C will be zero?



Solution:

Remember electric field is a vector quantity.

a) Vector sum of the fields at point C:

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

(It is easy! to see that the first charge will be positive and second charge will be negative)

$$\vec{E}_1 = k\frac{Q_1}{a^2}\hat{x}; \qquad \vec{E}_2 = k\frac{Q_2}{\sqrt{2}a^2}(-\hat{x}-\hat{y})$$

Since **E** has no *x* component, then $Q_1 = Q_2/\sqrt{2}$.

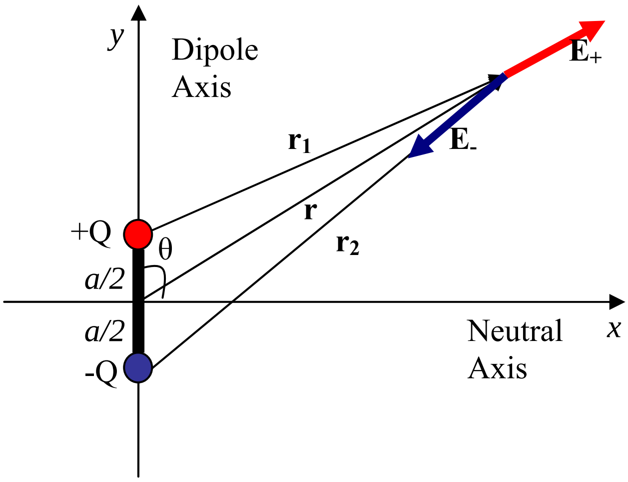
On the other hand according to the coulomb law

$$\vec{F} = k\frac{Q_1Q_2}{a^2}\hat{y}; \quad F = \sqrt{2}\text{N then } Q_1Q_2 = \sqrt{2}a^2/k$$

I thin you can solve the remaining….

Example 3

Electric field of a dipole moment. An electric dipole is a system of two equal and opposite point charges separated by a small distance. The extended straight line joining the two point charges in a dipole is called the dipole axis. The perpendicular bisector of the dipole axis is called the neutral axis.



The question is: calculate electric field at a distance **r** from the center of the dipole. The answer of this question is very easy. From the definition of electric field, we calculate

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1^2}\hat{r}_1 - \frac{1}{r_2^2}\hat{r}_2\right)$$

In practice *a* is a very small quantity when it compared to *r*. Distance *a* is in atomic scale. Therefore it is necessary to make an approximation ($r \gg a$). The calculation is tedious. Let us write the vectors:

$$\vec{r}_1 = \vec{r} - \frac{a}{2}\hat{y}; \quad \vec{r}_2 = \vec{r} + \frac{a}{2}\hat{y}.$$

Magnitude of the vectors can be calculated from the dot product property:

$$r_1^2 = r^2 + \left(\frac{a}{2}\right)^2 - ar\cos\theta; \quad r_2^2 = r^2 + \left(\frac{a}{2}\right)^2 + ar\cos\theta$$

and the unit vectors: $\hat{r}_1 = \frac{\vec{r}_1}{r_1}; \quad \hat{r}_2 = \frac{\vec{r}_2}{r_2}$

The electric field takes the form:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\vec{r} - \frac{a}{2}\hat{y}}{\left(r^2 + \left(\frac{a}{2}\right)^2 - ar\cos\theta\right)^{3/2}} - \frac{\vec{r} + \frac{a}{2}\hat{y}}{\left(r^2 + \left(\frac{a}{2}\right)^2 + ar\cos\theta\right)^{3/2}}\right)$$

The calculation can be completed later. (Potential section). Here we calculate specific form of the field.

Electric field on the dipole axis: To calculate electric field on the dipole axis we set θ=0 and $\vec{r} = y\hat{y}$, y is the distance from the origin to a point on the positive y-axis.

The electric field takes the form:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\left(y - \frac{a}{2}\right)\hat{y}}{\left(y - \left(\frac{a}{2}\right)\right)^3} - \frac{\left(y + \frac{a}{2}\right)\hat{y}}{\left(y + \left(\frac{a}{2}\right)\right)^3}\right)$$

using the identity $(1+s)^n \approx 1 + ns$ when $s \ll 1$ we obtain:

$$\vec{E} = \frac{Qa}{2\pi\varepsilon_0 y^3}\hat{y}$$

We define the dipole moment $\vec{p} = Qa\hat{y}$ ^which is directed from negative to positive charge.

Then the electric field: $\vec{E} = \frac{\vec{p}}{2\pi\varepsilon_0 y^3}$.

Electric field on the neutral axis: To calculate electric field on the neutral axis we set $\theta = \pi/2$ and $\vec{r} = x\hat{x}$, x is the distance from the origin to a point on the positive x-axis.

The electric field takes the form:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{x\hat{x} - \frac{a}{2}\hat{y}}{\left(x^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}} - \frac{x\hat{x} + \frac{a}{2}\hat{y}}{\left(x^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-a\hat{y}}{\left(x^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}}\right)$$

using the identity $(1+s)^n \approx 1 + ns$ when $s \ll 1$ we obtain:

$$\vec{E} = -\frac{Qa}{2\pi\varepsilon_0 x^3}\hat{y}$$

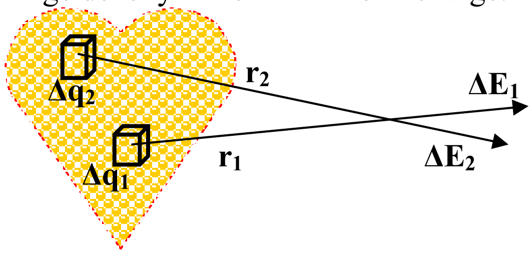
We define the dipole moment $\vec{p} = Qa\hat{y}$ ^which is directed from negative to positive charge.

Then the electric field: $\vec{E} = -\frac{\vec{p}}{2\pi\varepsilon_0 x^3}$.

Electric field of continuous charge distributions

What 's about continuous charge?

When we deal with continuous charge distributions, it is most convenient to express the charge on an object as a charge density rather than a total charge.



The total electric field is just the sum of the fields of the small (point) charges Δq's.

$$\vec{E} = \sum \Delta\vec{E}_i = \sum k\frac{\Delta q_i}{r_i^2}\hat{r}_i$$

For maximum accuracy we want the Δq's to become smaller and smaller. In this limit, the sum becomes an integral:

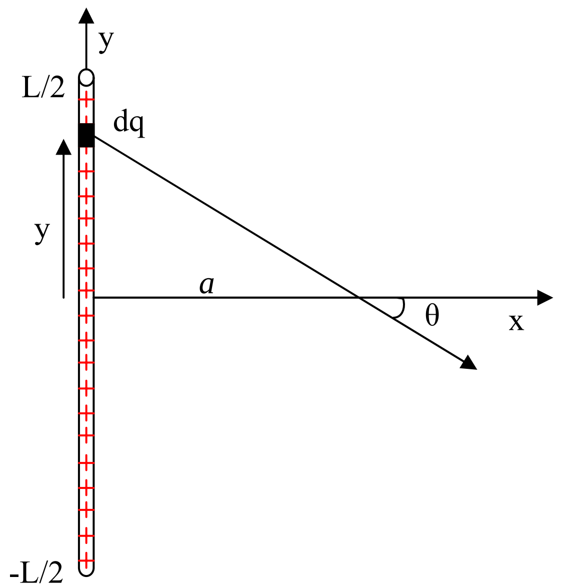
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

continuous charge distribution

Interpretation of *dq*
For a line of charge, for example, we would report the linear charge density (or charge per length) λ (C/m). Table below shows the other charge densities we shall be using:

Name	Symbol	SI unit
charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

Example: Electric field for lines of uniform charge
Find the electric field at distance *a* m as in figure, from the line of charge with charge density λ C/m, length L.



Step
I: let dy be the length of an element
II:Relate the charge dq of the element to the length of the element with dq=λdy
III:Express the field $\vec{E}$ produced at given point by dq using field equation for charge distribution.
IV:If the charge on the line is positive, then at given point draw a vector $\vec{E}$ that points directly away from dq. If the charge is negative, draw the vector pointing directly toward dq. The distance from dq to the point is r!
V:Express r in other forms for integration (∫dx)

e.g. $r = \sqrt{a^2 + y^2}$

VI:Express $\vec{E}$ in components e.g., $\vec{E}_x$, $\vec{E}_y$

VII:Look for canceling and adding components

VIII:Transform (x,y) to (r, θ) or (r, θ) to (x,y)

IX: Finalize your answer

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dy}{a^2+y^2}\hat{r}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dy}{a^2+y^2}\cos\theta$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dy}{a^2+y^2}\sin\theta$$

substituting $\cos\theta = a/r$ and $\sin\theta = y/r$ we obtain

$$E_x = \frac{1}{4\pi\varepsilon_0}\int\frac{a\lambda dy}{\left(a^2+y^2\right)^{3/2}}$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\int\frac{y\lambda dy}{\left(a^2+y^2\right)^{3/2}}$$

using the identities: $\int\frac{ydy}{\left(a^2+y^2\right)^{3/2}} = -\frac{1}{\sqrt{y^2+a^2}}$ and $\int\frac{ady}{\left(a^2+y^2\right)^{3/2}} = \frac{y}{a\sqrt{y^2+a^2}}$

and taking the limits of the integrals from –L/2 to L/2 we obtain:

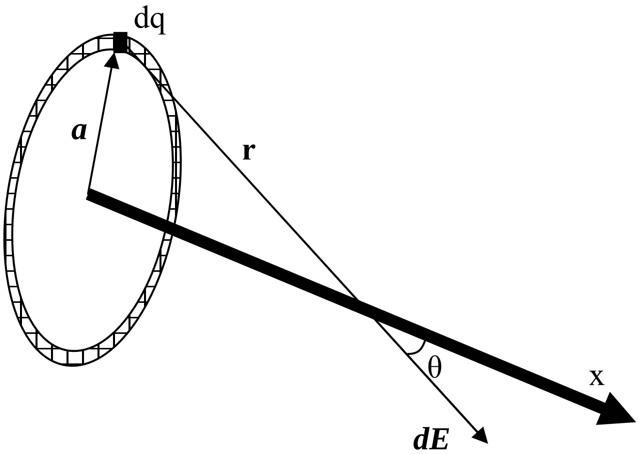
$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{a\sqrt{a^2+L^2/4}}$$

$$E_y = 0 \text{ (as expected)}$$

What is the electric field as L tends to infinity!

Example

Find the field on the axis of a ring of charge, q, (distributed uniformly over the ring) and radius, *a*, as a function of the distance from the center, x.



Using the field due to the point charge dq, $d\vec{E} = k\frac{dq}{r^2}\hat{r}$. By the symmetry of this problem the field will only point in the x-direction so we only have to worry about the x-components, $dE_x = k\frac{dq}{r^2}\cos\theta$.(You can find the electric field for y component, this will be zero). Notice that r is the same for all the dq's and cosθ is also the same for all dq's so the integral is straight forward.

$$E_x = k\int\frac{dq}{r^2}\cos\theta = k\frac{\cos\theta}{r^2}\int dq = k\frac{q}{r^2}\cos\theta$$

Note that $r^2 = a^2 + x^2$ and $\cos\theta = \frac{x}{\sqrt{a^2 + x^2}}$ so we can write $E_x = k\frac{q\ x}{\left(a^2 + x^2\right)^{\frac{3}{2}}}$.

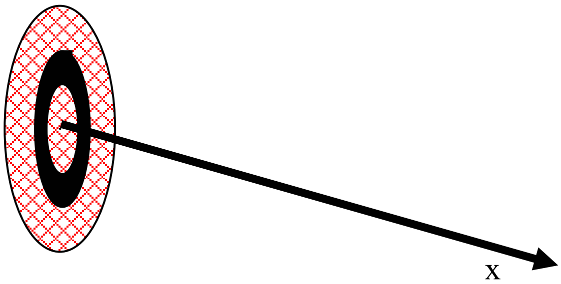
Check the answer by examining the limits as x goes to zero and to infinity.
What happens if *a* goes to zero! ☺

Example

Find the electric field a distance, x, away from disc of radius R and charge density, $\sigma(^{C}/_{m^2})$.

Solution: In the previous example we have obtained electric field of a ring. We can use the result to obtain field of the disc. Field of the ring of radius *a* is given by

$$E_x = k\frac{q\ x}{\left(a^2 + x^2\right)^{\frac{3}{2}}}.$$



We think that disc contains large number of rings of radius changing from zero to R. Electric field of each ring is directed along x-axis.

Here let us obtain the electric field by direct integration. Because of symmetry the electric field directed along x-axis. We can write:

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{xdq}{\left(\rho^2 + x^2\right)^{3/2}}; \quad dq = \sigma\rho d\rho d\phi$$

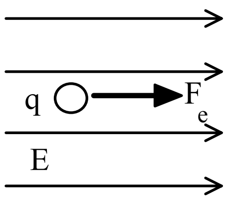
$d\rho$ is thickness of each ring. Integration yields:

$$E_x = \frac{\sigma x}{4\pi\varepsilon_0}\int_0^R \frac{2\pi\rho d\rho}{\left(\rho^2 + x^2\right)^{3/2}} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right).$$

When R tends to infinity we obtain electric field of a large conductor with surface charge density $\sigma(^{C}/_{m^2})$.

$$E_x = \frac{\sigma}{2\varepsilon_0}$$

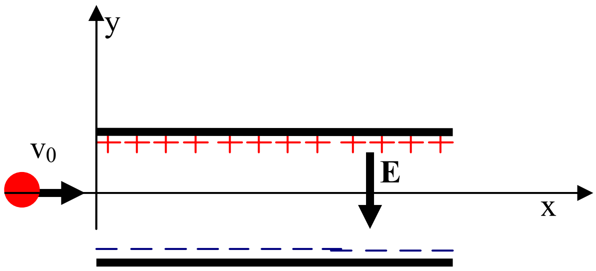
Motion of the Charges in Electric Fields



This section can be recognized as the application of the Newton's laws. Please review the kinematics, force, energy etc. sections in EP 105. The force on a single charge, as shown at the right, can now be written in terms of the field it feels. According to the definition of electric field, $\vec{E} \equiv \frac{\vec{F}_e}{q} \Rightarrow \vec{F}_e = q\vec{E}$.

Example

Figure shows an electron of mass m and charge e projected with speed v_0 at right angles to a uniform field **E**. Describe its motion.

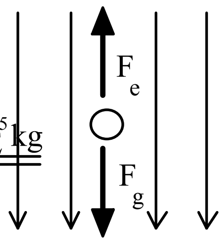


Remember projectile motion!!!

$$x = v_0 t \text{ and } y = \frac{1}{2}at^2 = \frac{eE}{2m}t^2 \text{ since } F = ma = eE$$

Eliminating t yields $y = \frac{eE}{2mv_0^2}x^2$ for the equation of trajectory.

Example:
The Earth has an electric field of about 150N/C pointed downward. A 1.00μm radius water droplet is suspended in calm air. Find (a)the mass of the water droplet, (b)the charge on the water droplet and (c)the number of excess electrons on the droplet.



(a)Use the definition of density and the volume of a sphere,

$$\rho \equiv \frac{m}{vol} \Rightarrow m = \rho(vol) = \rho \tfrac{4}{3}\pi r^3 = (1000)\tfrac{4}{3}\pi(1.00x10^{-6})^3 = \underline{\underline{4.19x10^{-15}\,kg}}$$

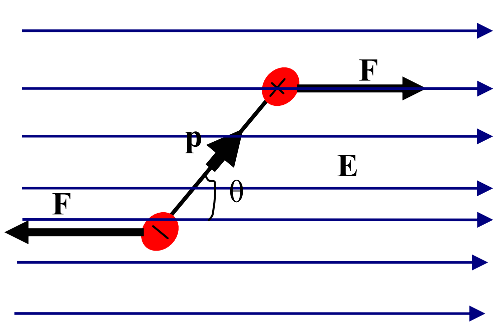
(b)The forces on the droplet are its weight and the electric force. Using Newton's Second Law, $\Sigma F = ma \Rightarrow F_e - F_g = 0 \Rightarrow F_e = F_g$

Using the definitions of the electric and gravitational fields,

$$qE = mg \Rightarrow q = \frac{mg}{E} = \frac{(4.19x10^{-15})(9.80)}{150} = \underline{\underline{2.74x10^{-16}C}}$$

(c)Since charge is quantized, $q = Ne \Rightarrow N = \frac{q}{e} = \frac{2.74x10^{-16}}{1.60x10^{-19}} = \underline{\underline{1710 \text{ electrons}}}$

Example



A dipole in a constant electric field as shown at the left. It feels no net force because the two forces on it caused by the field are equal and opposite. The dipole does feel a net torque, however. This torque tends to align it with the field.

$$\sum \tau = (a\sin\theta)(qE) + (a\sin\theta)(qE) = 2aqE\sin\theta = pE\sin\theta$$

This torque points into the paper so we can write the torque on the dipole as,

$\boxed{\vec{\tau} = \vec{p} \times \vec{E} \quad \text{The Torque on a Dipole}}$

The potential energy of the dipole can be found from the definition of potential energy, $\Delta U \equiv -W_c$.

The work done as the dipole rotates through an angle, dθ, is, $dW = \tau\, d\theta = -pE\sin\theta\, d\theta$.

The total work done as the angle goes from π/2 to θ is, $W = -\int_{\pi/2}^{\theta} pE\sin\theta\, d\theta = pE\cos\theta$.

The potential energy is $\Delta U = U(\theta) - U(\pi/2) = -pE\cos\theta \Rightarrow U = -\vec{p}\bullet\vec{E}$, where the zero for potential energy is $\theta = \pi/2$.

$\boxed{U = -\vec{p}\bullet\vec{E} \quad \text{The Potential Energy of a Dipole}}$

Example
Water molecules have a dipole moment of 6.20×10^{-30} C·m. Find (a)the maximum torque on a water molecule in the E-field of Earth and (b)the potential energy lost as the molecule moves from the position of maximum torque until it aligns with the field.

(a)The torque on a dipole in a constant field is $\vec{\tau} = \vec{p} \times \vec{E}$. The maximum occurs when the dipole moment is perpendicular to the field, $\tau = pE = (6.20 \times 10^{-30})(150) = \underline{\underline{9.30 \times 10^{-28} \mathrm{N \cdot m}}}$

(b)The potential energy of a dipole is $U = -\vec{p} \bullet \vec{E}$. When the moment is perpendicular to the field this is zero. When the moment is aligned with the field $U = -pE = -9.30 \times 10^{-28}\,\mathrm{J}$. This then is the energy that is lost $U_{lost} = \underline{\underline{9.30 \times 10^{-28}\,\mathrm{J}}}$.