



Date: 02/05/2019 Time: 10:30 Duration: 90 min.

EDUCATION : 1st Ed. 2nd Ed.
DEPARTMENT : CE MME IE ME TE

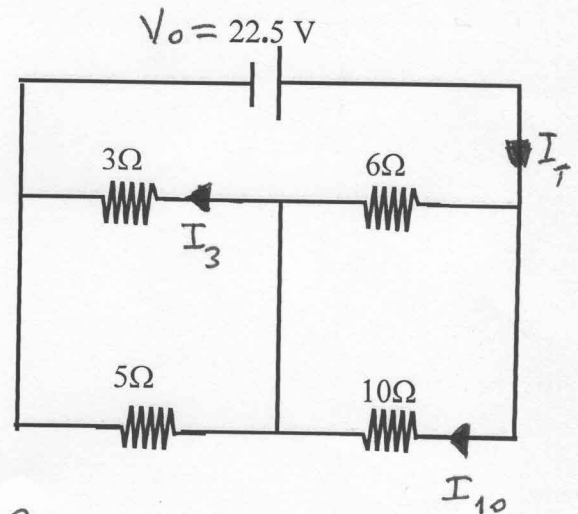
Name	Surname	Student No	Signature
	S O L U T I O N S		

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- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants: $\pi=3.14$, $k=9 \times 10^9 \text{ N.m}^2/\text{C}^2$, $\epsilon_0=8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$, $m_e=9.1 \times 10^{-31} \text{ kg}$, $|e|=1.6 \times 10^{-19} \text{ C}$, $\mu_0=4\pi \times 10^{-7} \text{ T.m/A}$
- $1 \text{ mm}=10^{-3} \text{ m}$, $1 \text{ cm}=10^{-2} \text{ m}$, $1 \text{ nm}=10^{-9} \text{ m}$, $1 \text{ pm}=10^{-12} \text{ m}$, $1 \text{ h}=3600 \text{ s}$, $1 \text{ min}=60 \text{ s}$, $1 \text{ rev}=2\pi \text{ rad}$.

QUESTION 1 (20 %)

Find the voltage across and current through 3Ω and 10Ω resistors in the given figure.



- 3Ω and 5Ω are parallel to each other

$$R_{3,5} = \frac{3 \times 5}{3 + 5} = 1.875 \Omega$$

- 6Ω and 10Ω are parallel to each other:

$$R_{6,10} = \frac{6 \times 10}{6 + 10} = 3.75 \Omega$$

- Total (equivalent resistance) = $R_{3,5} + R_{6,10} = 5.625 \Omega$

Total current in the circuit: $I_T = \frac{V_0}{R_{eq}} = \frac{22.5}{5.625} = 4 \text{ A}$.

Current divider rule:

$$I_{10} = \left(\frac{6}{6+10} \right) I_T = \frac{6}{16} (4) = 1.5 \text{ A} \Rightarrow V_{10} = I_{10} R_{10} = (1.5)(10) = 15 \text{ V}$$

$$I_3 = \left(\frac{5}{3+5} \right) I_T = \frac{5}{8} (4) = 2.5 \text{ A} \Rightarrow V_3 = (2.5)(3) = 7.5 \text{ V}$$

$I_3 = 2.5 \text{ A}$

$I_{10} = 1.5 \text{ A}$

$V_3 = 7.5 \text{ V}$

$V_{10} = 15.0 \text{ V}$

QUESTION 2 (20 %)

For the given circuit;

- (a) Find the capacitance value of third capacitor (C_3) if the distance between plates is 8.85 mm and area of each plate is $25 \times 10^{-4} \text{ m}^2$.

$$C_3 = \frac{\epsilon_0 A}{d} K$$

$$= \frac{(8.85 \times 10^{-12})(25 \times 10^{-4})}{8.85 \times 10^{-3}} (4)$$

$$= 10 \times 10^{-12} \text{ F} \Rightarrow C_3 = 10 \text{ pF}$$

$$C_3 = 10 \text{ pF}$$

- (b) Find the equivalent capacitance value between points "a" and "b".

$$C_{13} = \frac{C_1 C_3}{C_1 + C_3} = \frac{(10)(10)}{10 + 10} = 5 \text{ pF}$$

$$C_{eq} = C_{13} + C_2 = 5 + 5 = 10 \text{ pF}$$

$$C_{eq} = 10 \text{ pF}$$

- (c) Find the total charge on the second capacitor (C_2).

$$Q_2 = C_2 \Delta V = (5)(40) = 200 \text{ pC}$$

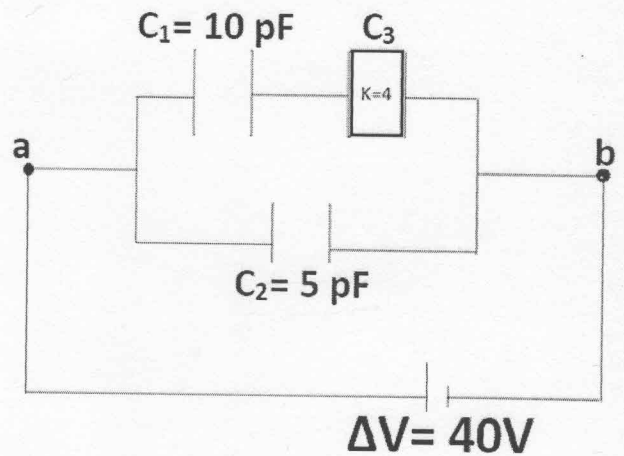
$$= 200 \times 10^{-12} \text{ C}$$

$$Q_2 = 200 \text{ pC}$$

- (d) Find the potential energy stored in the second charged capacitor (C_2).

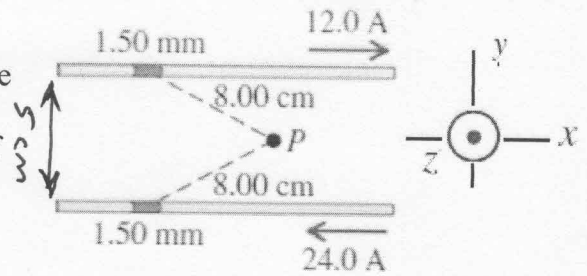
$$U_2 = \frac{Q_2^2}{2C_2} = \frac{(200 \times 10^{-12})^2}{(2)(5 \times 10^{-12})} = 4 \times 10^{-9} \text{ J}$$

$$U_2 = 4 \times 10^{-9} \text{ J}$$



QUESTION 3 (20 %)

Two parallel wires are 5 cm apart and carry currents in opposite directions, as shown. Find the magnitude and direction of the total magnetic field at point P due to two 1.50 mm segments of wire. [The distance from each segment to the point P is 8 cm].



magnetic field due to small current element is:

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl \sin \theta}{r^2}$$

$$dl = 1.5 \text{ mm}$$

$$\sin \theta = \frac{2.5}{8} = 0.3125$$

$$r = 8 \text{ cm}$$

Top wire:

$$dB_1 = \frac{4\pi \times 10^{-7} (12)}{4\pi} \frac{(1.5 \times 10^{-3}) (0.3125)}{(0.08)^2} = 8.79 \times 10^{-8} \text{ T } (\otimes)$$

Bottom wire:

$$dB_2 = \frac{4\pi \times 10^{-7} (24)}{4\pi} \frac{(1.5 \times 10^{-3}) (0.3125)}{(0.08)^2} = 1.75 \times 10^{-7} \text{ T } (\otimes)$$

$$\text{Total } B_{\pm} = dB_1 + dB_2 = 2.64 \times 10^{-7} \text{ T } (\otimes) \quad \boxed{dB_{\pm} = 2.64 \times 10^{-7} \text{ T } (\hat{-z})}$$

QUESTION 4 (20 %)

A cylindrical copper wire having diameter 6 mm and length 20 m carries a current of 8 A. The wire is initially operating at room temperature at $T = 20^\circ\text{C}$.

(a) What is the current density in the wire?

$$D = 6 \text{ mm} \Rightarrow r = 3 \text{ mm}$$

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{8}{\pi (3 \times 10^{-3})^2} = 2.8 \times 10^5 \text{ A/m}^2$$

$$= 2.8 \times 10^5 \frac{\text{A}}{\text{mm}^2} \quad \boxed{J = 2.8 \times 10^5 \text{ A/m}^2}$$

(b) Find the drift speed of the electrons in the wire if there are $n = 8.9 \times 10^{28}$ electrons/ m^3 .

$$v_d = \frac{J}{ne} = \frac{2.8 \times 10^5}{(8.9 \times 10^{28})(1.6 \times 10^{-19})} = 2 \times 10^{-5} \text{ m/s}$$

$$\boxed{v_d = 2 \times 10^{-5} \text{ m/s}}$$

(c) What is the resistance of the wire at $T = 100^\circ\text{C}$ [$\alpha = 3.9 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$, $\rho = 1.7 \times 10^{-8} \text{ } \Omega \cdot \text{m}$ at $T = 20^\circ\text{C}$]

Resistance at $T = 20^\circ\text{C}$

$$R = \rho \frac{L}{A} = (1.7 \times 10^{-8}) \left(\frac{20}{\pi (3 \times 10^{-3})^2} \right) = 1.2 \times 10^{-2} \text{ } \Omega$$

$$= 0.012 \text{ } \Omega$$

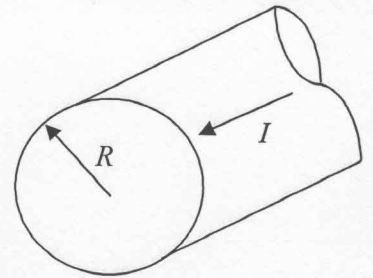
at $T = 100^\circ\text{C}$

$$R = R_0 [1 + \alpha (T - T_0)] \quad \boxed{R_{100} = 0.016 \text{ } \Omega}$$

$$= 0.012 [1 + 3.9 \times 10^{-3} (100 - 20)] = 0.0157 \text{ } \Omega$$

QUESTION 5 (20 %)

A long straight cylindrical conducting wire of radius R carries a current I of a uniform current density.



(a) Determine the magnetic field at a radial distance r from the axis of the wire for $r > R$.

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad r > R$$



$$B = \mu_0 I / 2\pi r$$

(b) Determine the magnetic field at a radial distance r from the axis of for $r < R$.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 \frac{\pi r^2}{\pi R^2} I$$

$$B = \frac{\mu_0 I}{2\pi} \left(\frac{r}{R^2} \right) \quad r < R$$



$$B = \mu_0 I r / (2\pi R^2)$$

(c) Calculate the magnetic fields numerically at $r = 5 \text{ mm}$ and $r = 2 \text{ mm}$ if the radius of the wire is $R = 3.5 \text{ mm}$ and the current is $I = 25 \text{ A}$.

$$B(5 \text{ mm}) = \frac{(4\pi \times 10^{-7})(25)}{(2\pi) 5 \times 10^{-3}} = 1 \times 10^{-3} \text{ T}$$

$$B(2 \text{ mm}) = \frac{(4\pi \times 10^{-7})(25)(2 \times 10^{-3})}{(2\pi)(3.5 \times 10^{-3})^2} = 8.16 \times 10^{-4} \text{ T}$$

$$B = \begin{cases} \frac{\mu_0 I}{2\pi r} & r > R \\ \frac{\mu_0 I r}{2\pi R^2} & r < R \end{cases}$$

$$B_{out} = 1 \times 10^{-3} \text{ T}$$

$$B_{in} = 8.16 \times 10^{-4} \text{ T}$$