



Date: 21/03/2019 Time: 10:30 Duration: 90 min.

Ques.	Mark
1	
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Total	

EDUCATION : 1st Ed. 2nd Ed.
DEPARTMENT : CE MME IE ME TE

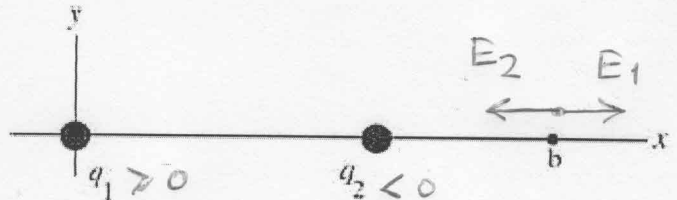
Name	Surname	Student No	Signature

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants: $\pi=3.14$, $k = 9 \times 10^9 \text{ N.m}^2/\text{C}^2$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$, $m_e = 9.1 \times 10^{-31} \text{ kg}$, $|e| = 1.6 \times 10^{-19} \text{ C}$, $\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A}$
- $1 \text{ mm} = 10^{-3} \text{ m}$, $1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ nm} = 10^{-9} \text{ m}$, $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ h} = 3600 \text{ s}$, $1 \text{ min} = 60 \text{ s}$, $1 \text{ rev} = 2\pi \text{ rad}$.

QUESTION 1 (20 %)

A point charge $q_1 = +2 \text{ nC}$ is at the origin ($x = 0$), and a second point charge $q_2 = -5 \text{ nC}$ is on the x-axis at $x = 0.7 \text{ m}$.

(a) Find the electric field (magnitude and direction) at $x = 1.2 \text{ m}$ (point b) on the x-axis



$$E_{1x} = \frac{k |q_1|}{r_1^2} = 9 \times 10^9 \frac{2 \times 10^{-9}}{(1.2)^2} = 12.5 \text{ N/C } +x \text{ direction}$$

$$E_{2x} = \frac{k |q_2|}{r_2^2} = 9 \times 10^9 \frac{5 \times 10^{-9}}{(0.5)^2} = 180 \text{ N/C } -x \text{ direction}$$

$$E_x = E_{1x} + E_{2x} = 12.5 - 180 = -167.5 \text{ N/C } -x \text{ dir.}$$

$$E_y = 0$$

$$E_z = 0$$

$$E = 167.5 \text{ N/C}$$

$$-x \text{ direction}$$

(b) Find the net electric force that the two charges would exert on an electron placed at point b.

Force acting on electron is

$$\vec{F} = -e\vec{E}$$

$$= (-1.6 \times 10^{-19} \text{ e}) (-167.5 \hat{i} \text{ N/C})$$

$$= 2.68 \times 10^{-17} \text{ N } (+x \text{ dir.})$$

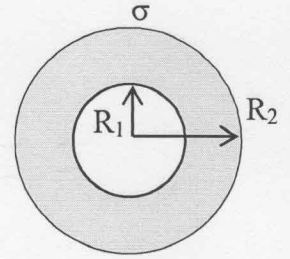
$$\vec{F} = 2.68 \times 10^{-17} \hat{i} \text{ (N)}$$

QUESTION 2 (20 %)

Charge Q is uniformly distributed on a conducting spherical shell. The surface charge density is given by $\sigma = +8.0 \times 10^{-6} \text{ C/m}^2$. Inner and outer radius of the shell are respectively given by $R_1 = 5 \text{ cm}$ and $R_2 = 10 \text{ cm}$.

(a) What is the total charge (Q) on the surface of the sphere?

$$\begin{aligned}
 Q &= \sigma A = \sigma 4\pi R_2^2 \\
 &= \left(8 \times 10^{-6} \frac{\text{C}}{\text{m}^2}\right) \left(4\pi (0.1 \text{ m})^2\right) \\
 &= 1 \times 10^{-6} \text{ C} \\
 &= 1 \mu\text{C}
 \end{aligned}$$



$$Q = 1 \times 10^{-6} \text{ C}$$

(b) What is the electric field at $r = 3 \text{ cm}$ where r is the distance from the center of the sphere?

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

$$E = 0 \text{ since } q_{enc} = 0$$



$$E = 0$$

(c) What is the electric field at $r = 8 \text{ cm}$ where r is the distance from the center of the sphere?

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

$$E = 0 \text{ since } q_{enc} = 0 \text{ inside the conductor.}$$



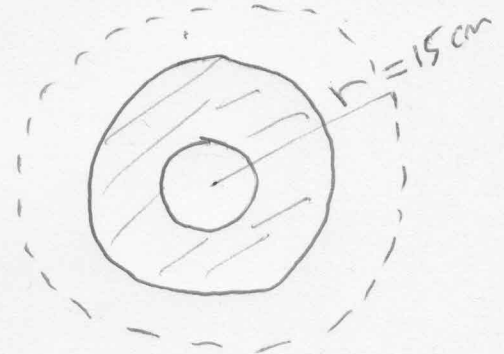
$$E = 0$$

(d) What is the electric field at $r = 15 \text{ cm}$ where r is the distance from the center of the sphere?

$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\begin{aligned}
 E &= \frac{Q}{4\pi\epsilon_0 r^2} = k \frac{Q}{r^2} \\
 &= (9 \times 10^9) \frac{1 \times 10^{-6}}{(0.15)^2} \\
 &= 4 \times 10^5 \text{ N/C}
 \end{aligned}$$

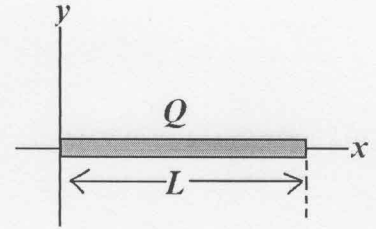


$$E = 4 \times 10^5 \text{ N/C}$$

QUESTION 3 (20 %)

A thin rod of length $L = 1$ m lies along the x axis with its left end at the origin. It has a uniform linear charge distribution $\lambda = -3.2$ C/m.

How many electrons are there on the rod contributing to the total charge?



$$\text{charge density} = \frac{\text{charge}}{\text{length}}$$

$$\lambda = \frac{Q}{L}$$

$$\Rightarrow Q = \lambda L = (-3.2 \frac{\text{C}}{\text{m}})(1\text{m}) = -3.2 \text{ C}$$

Number of electronic charges can be found from quantization.

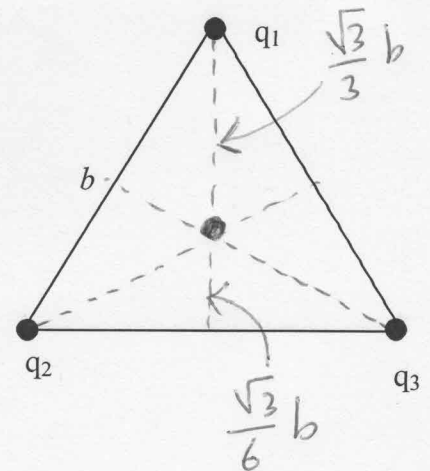
$$n = \frac{Q}{e} = \frac{-3.2 \text{ C}}{-1.6 \times 10^{-19} \text{ C}} = 2 \times 10^{19} \text{ electrons}$$

$$n = 2 \times 10^{19}$$

QUESTION 4 (20 %)

Three particles ($q_1 = +2$ nC, $q_2 = -2$ nC, $q_3 = +2$ nC) are located at the corner of an equilateral triangle of side $b = 20$ cm.

Find the electric potential at the center of the triangle.



$$V = V_1 + V_2 + V_3$$

$$= \frac{k q_1}{\frac{\sqrt{3}}{3} b} + \frac{k q_2}{\frac{\sqrt{3}}{3} b} + \frac{k q_3}{\frac{\sqrt{3}}{3} b}$$

$$= \frac{k}{\frac{\sqrt{3}}{3} b} (q_1 + q_2 + q_3)$$

$$= \frac{k}{\frac{\sqrt{3}}{3} b} (q - q + q)$$

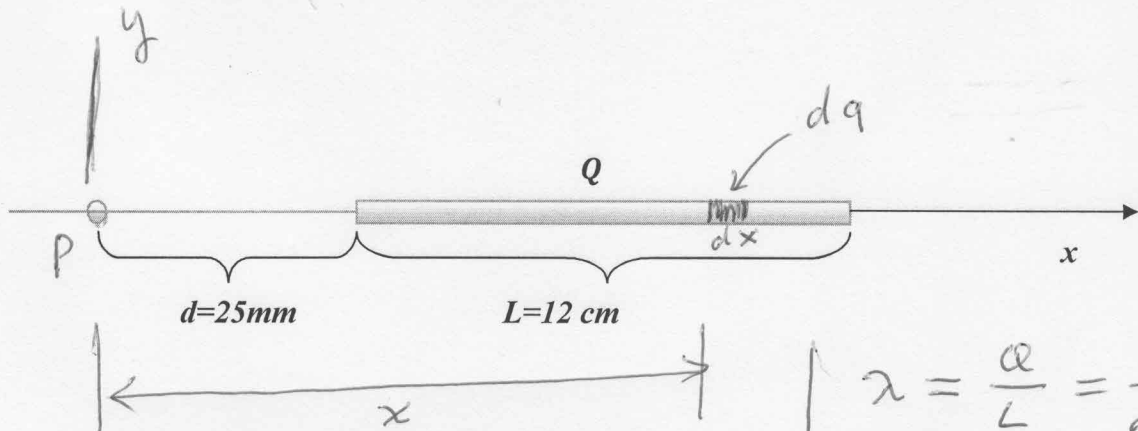
$$= \frac{k q}{\frac{\sqrt{3}}{3} b}$$

$$= \frac{(9 \times 10^9)(2 \times 10^{-9})}{\frac{\sqrt{3}}{3} (0.2)} = 155.88 \text{ Volt}$$

$$V = 155.88 \text{ V}$$

QUESTION 5 (20 %)

Figure shows a thin plastic rod of length $L = 12$ cm and uniform positive charge $Q = 56.1$ fC lying on an x axis ($1\text{fC} = 1 \times 10^{-15}$ C). If the potential at infinity is assumed to be zero, find the electric potential at point P on the axis, at distance $d = 25$ mm from the rod.



Electric potential at point P :

$$\lambda = \frac{Q}{L} = \frac{dq}{dx}$$

$$\text{or } dq = \lambda dx = \frac{Q}{L} dx$$

$$V = k \int \frac{dq}{r}$$

$$= k \int_d^{d+L} \frac{\lambda dx}{x}$$

$$= k \lambda [\ln(d+L) - \ln(d)]$$

$$= k \frac{Q}{L} \left[\ln\left(\frac{d+L}{d}\right) \right]$$

$$= 9 \times 10^9 \frac{56.1 \times 10^{-15}}{0.12} \left(\ln\left(\frac{0.025 + 0.12}{0.025}\right) \right)$$

$$= 7.4 \times 10^{-3} \text{ Volt}$$

$$\boxed{V = 7.4 \times 10^{-3} \text{ V}}$$