UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS
EP 106 General Physics II
Second Midterm Exam *SPRING SEMESTER*

*** SOLVE ONLY 5 OUT OF 6 QUESTIONS ***

Date: 24/05/2007 **Time:** 110 min.

Name	Surname	Student No	Dep.	Signature
— SOLUTIONS —				

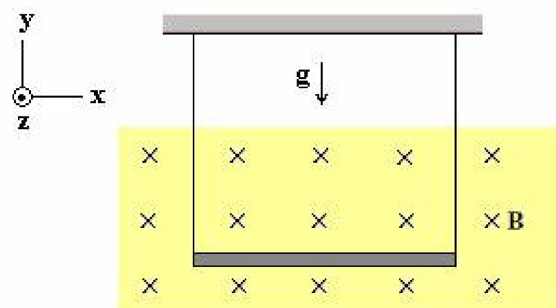
Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
OUT OF	100

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $e = 1.6\times10^{-19}$ C, $\varepsilon_0 = 8.85\times10^{-12}$ C^2/N·m^2, $\mu_0 = 4\pi\times10^{-7}$ T·m/A, $g=9.8$ m/s^2

QUESTION 1 (20 %)

A conducting wire, suspended by two flexible strings as shown in the figure, has a mass per unit length of 0.04 kg/m. The wire is placed in a region containing uniform magnetic field of $B = 3.6$ T into page.

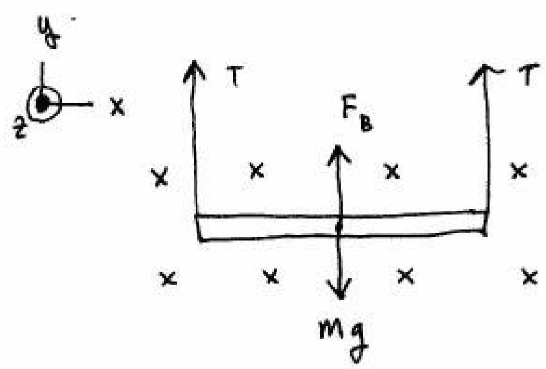
What current (magnitude and direction) must exist in the wire for the tension in the supporting strings to be zero?



Equilibrium condition:

$$\underbrace{2\vec{T}}_{0} + \vec{F}_B + m\vec{g} = 0$$

or $$\vec{F}_B = -m\vec{g} = mg\hat{j}$$



$\therefore$ Direction of F_B is $+y$.
From right hand rule direction of the current is $+x$.

$$\mu = \frac{m}{L} = 0.04 \text{ kg/m}$$

$$\vec{F}_B = i\vec{L}\times\vec{B} = iL\hat{i}\times(-B\hat{k}) = iLB(-\hat{i}\times\hat{k}) = iLB\hat{j}$$

$$F_B = mg$$

$$iLB = mg \rightarrow i = \frac{mg}{LB} = \mu g/B = (0.04)(9.8)/(3.6)$$
$$= 0.11 \text{ A}$$

$i =$ 0.11 A	In direction of +X

QUESTION 2 (20 %)

A current carrying aluminum wire, with a diameter of D = 0.10 mm and length L = 2 m, has a uniform electric field of E = 0.2 V/m imposed along its entire length. The temperature of the wire is T = 50 °C. Assume one free electron per atom.
(T_0=20 °C, $\rho_0 = 2.82\text{x}10^{-8}$ Ω·m, α = $3.9\text{x}10^{-3}$ 1/°C. Take the atomic mass of the Al as m_A =27 g/mole, density of the Al d = 2.7 g/cm^3 and the Avogadro's number N_A = $6.02\text{x}10^{23}$ atoms/mole)

(a) Determine the resistivity of the wire.

$$\rho = \rho_0[1+\alpha(T-T_0)]$$
$$= 2.82\times10^{-8}[1+3.9\times10^{-3}(50-20)]$$
$$= 3.15\times10^{-8}\ \Omega\cdot m$$

$\boxed{\rho = 3.15\times10^{-8}\ \Omega\cdot m}$

(b) What is the current density in the wire?

$$J = \frac{E}{\rho} = \frac{0.20}{3.15\times10^{-8}} = 6.35\times10^{6}\ A/m^2$$

$\boxed{J = 6.35\times10^{6}\ \frac{A}{m^2}}$

(c) What is the total current in the wire?

$$i = JA = J\left(\frac{\pi D^2}{4}\right)$$
$$= 6.35\times10^{-8}\left(\frac{\pi(0.1\times10^{-3})^2}{4}\right)$$
$$= 49.9\times10^{-3}\ A \approx 50\ mA$$

$\boxed{i = 50\times10^{-3}\ A}$

(d) What is the drift speed of the conduction electrons?

Number of free electrons per unit volume can be found from:

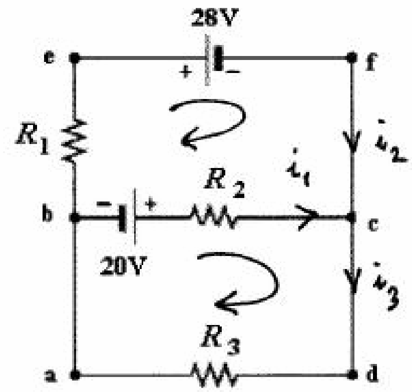
$$n = \frac{d N_A}{m_A} \quad \text{(see lecture notes)}$$

$$n = \frac{2.7\ g/cm^3}{27\ g/mole}\left(6.02\times10^{23}\ \frac{atoms}{mole}\right) = 6.02\times10^{22}\ e/cm^3$$
$$= 6.02\times10^{28}\ e/m^3$$

$$v_d = \frac{J}{ne} = \frac{6.35\times10^{6}}{(6.02\times10^{28})(1.6\times10^{-19})} = 0.66\times10^{-3}\ m/s$$
$$= 0.66\ mm/s$$

$\boxed{v_d = 0.66\times10^{-3}\ \frac{m}{s}}$

QUESTION 3 (20 %)



The values of the resistances in the circuit shown in the figure are given by

$R_1 = 4\ \Omega$
$R_2 = 6\ \Omega$
$R_3 = 2\ \Omega$

(a) Find the current on each resistance by *using Kirchoff's Rules*

Junction rule at point c:

$$i_3 = i_1 + i_2 \quad (1)$$

Loop abcda:

$$20 - 6\,i_1 - 2\,i_3 = 0 \quad (2)$$

Loop befcb:

$$-4i_2 - 28 + 6\,i_1 - 20 = 0 \quad (3)$$

Solving i_1, i_2 and i_3 from equations (1), (2) and (3)

$i_1 = 4A$
$i_2 = -6A$
$i_3 = -2A$

directions must be opposite

$i_1 =$	$4A$
$i_2 =$	$6A$
$i_3 =$	$2A$

(b) Find the potential difference between **c** and **b** (V_{bc}).

$V_c = 6\,i_1 - 20 + V_b$ or $V_c - 2\,i_3 = V_b$

$V_c - V_b = 2(2) = +4\ V$

$V_c - V_b = 6(4) - 20$

$= +4\ V$

$V_{bc} = +4V$

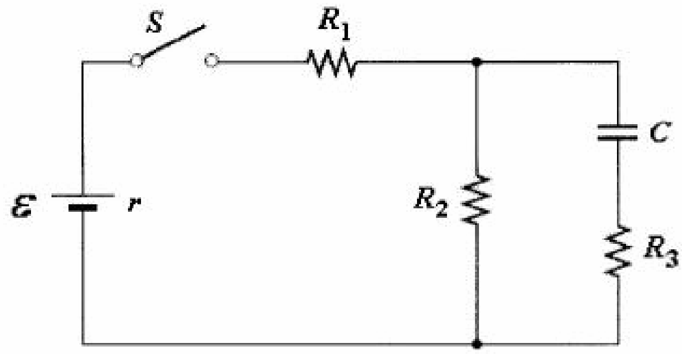
(c) Calculate the power delivered to 4Ω-resistor.

$$P = i_2^2\,R_1 = (6)^2\,4 = 144\ W$$

$P = 144\ W$

QUESTION 4 (20 %)

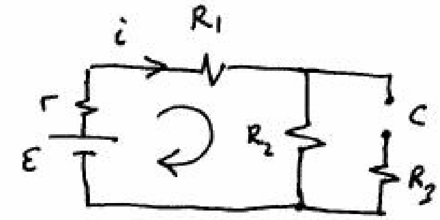
In Figure, assume that $R_1 = 12\ \Omega$, $R_2 = 15\ \Omega$ $R_3 = 5\ \Omega$, $C = 250\ \mu F$ and $\mathcal{E} = 9$ V. The battery has an internal resistance of $r = 3\ \Omega$.



Suppose that the switch has been closed for a length of time sufficiently long for the capacitor to become fully charged.

(a) Find the steady-state current in each external resistor.

when $t \to \infty$ capacitor behaves like an open circuit.



$$\therefore \quad i_3 = 0$$

$$i_1 = i_2 = \frac{\mathcal{E}}{R_1 + R_2 + r} = \frac{9}{12 + 15 + 3} = 0.3\ A$$

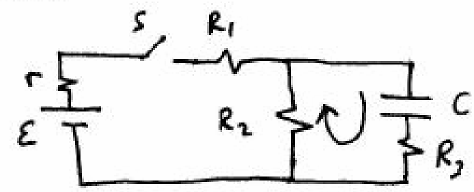
$i_1 =$ 0.3 A	$i_2 =$ 0.3 A	$i_3 =$ 0

(b) Find the charge on the capacitor.

$$Q = C\mathcal{E} = (250 \times 10^{-6})(9) = 2.25 \times 10^{-3} C$$

$\boxed{Q = 2.25 \times 10^{-3}\ C}$

(c) The switch is now opened at $t = 0$. Determine the current in R_2 at $t = 5$ ms.



time constant: $\tau = (R_2 + R_3) C$

$$= (15 + 5)(250 \times 10^{-6})$$

$$= 5 \times 10^{-3}\ s = 5\ ms$$

$$i_2 = i_3 = \frac{\mathcal{E}}{R_2 + R_3} e^{-t/\tau} = \frac{9}{15+5} e^{-5/5} = 0.17\ A$$

$\boxed{i = 0.17\ A}$

(d) Find the time that it takes for the charge on the capacitor to fall to one-fifth its initial value.

$$\frac{1}{5} Q_0 = Q_0 e^{-t/\tau}$$

$$\frac{1}{5} = e^{-t/\tau} \rightarrow t = \ln 5\ \tau$$

$$= (\ln 5)\ 5 \times 10^{-3}\ s$$

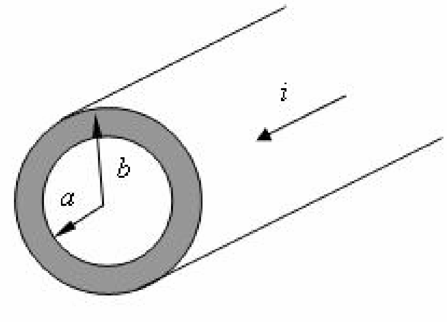
$$= 8.05 \times 10^{-3}\ s$$

$$= 8.05\ ms$$

$\boxed{t = 8.05 \times 10^{-3}\ s}$

QUESTION 5 (20 %)

Figure shows a section of a long hallow cylindrical conductor of radii a = 5 cm and b = 10 cm, carrying a uniformly distributed current i. The magnitude of the magnetic field on its outer surface, at $r = b$, is measured to be B = 0.2 T, where r is the radial distance from the cylindrical axis.



(a) Find the current in the wire?

Ampere's law on the outer surface $(r=b)$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc}$$

$$B(2\pi b) = \mu_0 i$$

$$i = \frac{2\pi B b}{\mu_0} = \frac{(2\pi)(0.2)(0.1)}{4\pi \times 10^{-7}} = 10^5 \text{ A}$$

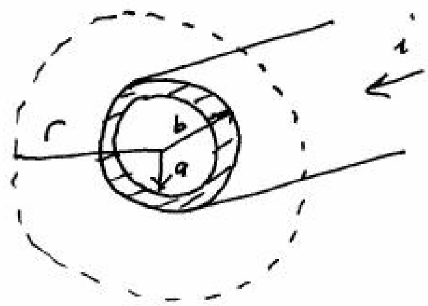
i = 10^5 A

(b) Find the magnitude of the magnetic field at r = 20 cm.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc}$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi \times 10^{-7})(10^5)}{(2\pi)(0.2)} = 0.1 \text{ T}$$

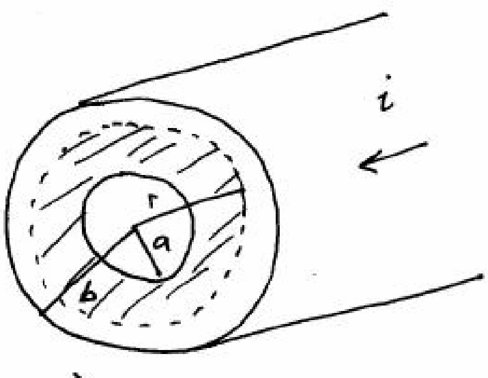


B = 0.10 T

(c) Find the magnitude of the magnetic field at r = 8 cm.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i_{enc}$$

$$B(2\pi r) = \mu_0 \left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2} \right) i$$



$$B = \frac{\mu_0 i}{2\pi} \left(\frac{r^2 - a^2}{r(b^2 - a^2)} \right)$$

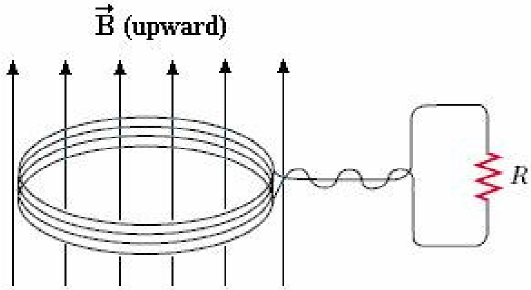
$$= \frac{(4\pi \times 10^{-7})(10^5)}{2\pi} \left(\frac{0.08^2 - 0.05^2}{0.08(0.10^2 - 0.05^2)} \right)$$

$$= 0.13 \text{ T}$$

B = 0.13 T

QUESTION 6 (20 %)

A circular coil enclosing an area of $A = 100\ \text{cm}^2$ is made of $N = 200$ turns of copper wire, as shown in the figure. The coil is placed in a region containing a magnetic field which is directed to the upward. The magnetic field is changed uniformly from $B = 0$ to $B = 1$ T in 4 seconds. Assume that resistance of the coil is negligible and external resistance is $R = 5\ \Omega$.



(a) Find the induced emf produced in the coil during the time interval of 4 s.

$$\frac{dB}{dt} = \frac{1\ T}{4\ s} = +0.25\ \ T/s$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA)$$

$$= -NA\frac{dB}{dt}$$

$$= -(200)(100\times10^{-4})(0.25)$$

$$= -0.5\ V$$

$\boxed{\varepsilon = -0.5\ V}$

(b) Find the magnitude of the current in the external resistor R during the period of 4 s.

$$i = \frac{|\varepsilon|}{R} = \frac{0.5\ V}{5\ \Omega} = 0.1\ A$$

$\boxed{i = 0.1\ A}$

(c) Find the amount of charge flowing through the coil during the period of 4 s.

$$Q = it = (0.1)(4) = 0.4\ C$$

$\boxed{Q = 0.4\ C}$