UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS
EP 106 General Physics II
First Midterm Exam *SPRING SEMESTER*

***** SOLVE ONLY 5 QUESTIONS OUT OF 6 *****

Date: 05/04/2007 Time: 110 min.

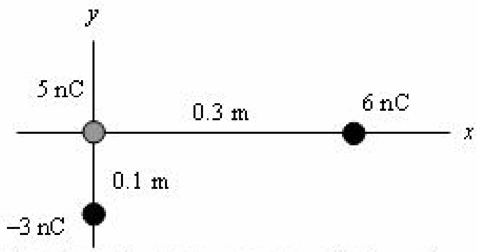
Name	Surname	Student No	Dep.	Signature
	Solutions !			

Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
OUT OF	100

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $e = 1.6\times10^{-19}$ C, $\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$, $k = 9\times10^9\ N\cdot m^2/C^2$

QUESTION 1 (20 %)

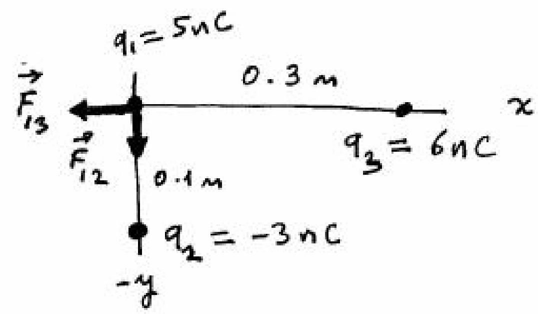
Three point charges are arranged as shown in the figure.
(Hint: $1\ nC = 10^{-9}$ C)



(a) Find *the Coulomb force* acting on the 5 nC charge in vector form due to presence of other charges.

$$\vec{F}_{13} = k\frac{q_1 q_3}{r_{13}^2}(-\hat{i}) = -9\times10^9\frac{(5\times10^{-9})(6\times10^{-9})}{(0.3)^2}\hat{i}$$

$$= -3\times10^{-6}\,\hat{i}\ (N)$$



$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}(-\hat{j}) = -9\times10^9\frac{(5\times10^{-9})(3\times10^{-9})}{(0.1)^2}\hat{j}$$

$$= -13.5\times10^{-6}\,\hat{j}\ (N)$$

$$\vec{F} = \vec{F}_{13} + \vec{F}_{12} = -3\times10^{-9}\,\hat{i} - 13.5\times10^{-6}\,\hat{j}\ (N)$$

$\boxed{\vec{F} = -3\times10^{-6}\,\hat{i} - 13.5\times10^{-6}\,\hat{j}}$ (N)

(b) Find *the magnitude* of that force

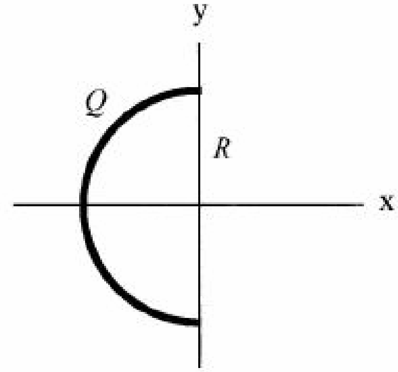
$$|\vec{F}| = \sqrt{F_{13}^2 + F_{12}^2} = \left[(-3\times10^{-6})^2 + (-13.6\times10^{-6})^2\right]^{1/2}$$

$$= 13.8\times10^{-6}\ N$$

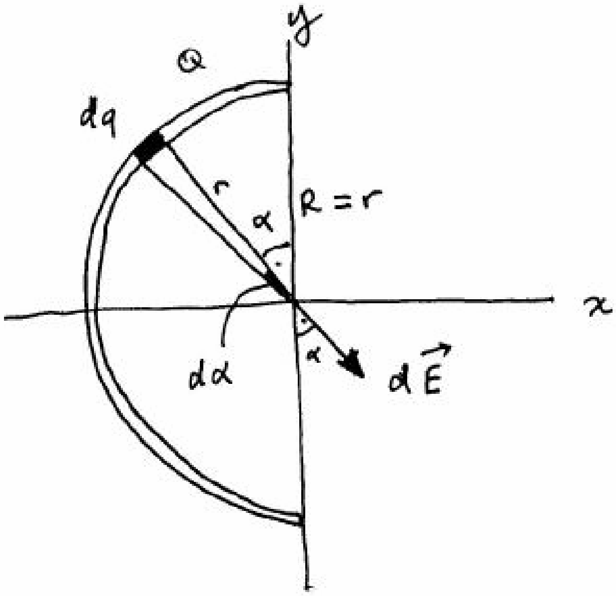
$\boxed{F = 13.8\times10^{-6}\ N}$

QUESTION 2 (20 %)

Charge Q is distributed uniformly along a rod. Then, this rod is bent to form a semicircle of radius R as shown in the figure.



Find an expression for the magnitude of the electric field at the center of the semicircle in terms of Q and R.



$$dq = \lambda \, d\ell = \left(\frac{Q}{\pi R}\right)(R \, d\alpha) = \frac{Q}{\pi} d\alpha$$

$$\vec{r} = R(\sin\alpha \, \hat{\imath} - \cos\alpha \, \hat{\jmath})$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = (\sin\alpha \, \hat{\imath} - \cos\alpha \, \hat{\jmath})$$

$$d\vec{E} = k \frac{dq}{r^2} \hat{r} = k \frac{(Q/\pi) \, d\alpha}{R^2} (\sin\alpha \, \hat{\imath} - \cos\alpha \, \hat{\jmath})$$

$$\vec{E} = \int d\vec{E} = \frac{kQ}{\pi R^2} \left\{ \int_0^{\pi} \sin\alpha \, d\alpha \, \hat{\imath} - \int_0^{\pi} \cos\alpha \, d\alpha \, \hat{\jmath} \right\}$$

$$= \frac{kQ}{\pi R^2} \left\{ -\cos\alpha \Big|_0^{\pi} \hat{\imath} - \sin\alpha \Big|_0^{\pi} \hat{\jmath} \right\}$$

$$= \frac{kQ}{\pi R^2} \left\{ 2\hat{\imath} + 0 \, \hat{\jmath} \right\}$$

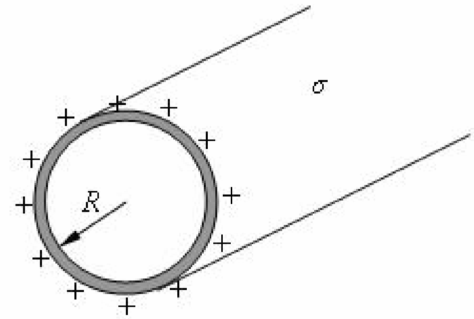
$$\therefore \vec{E} = \frac{2kQ}{\pi R^2} \hat{\imath} \Rightarrow E = \frac{2kQ}{\pi R^2}$$

$$\text{or} \quad E = \frac{Q}{2\pi^2 \varepsilon_0 R^2}$$

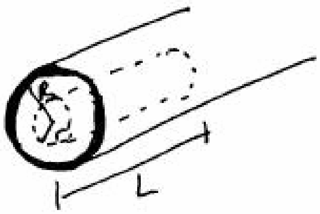
$E = 2kQ/\pi R^2$

QUESTION 3 (20 %)

Figure shows a section through a long, thin-walled metal tube of radius R, carrying a charge per unit area σ on its surface.



(a) Find an expression for the magnitude of electric field at a distance r from the tube axis where $r<R$



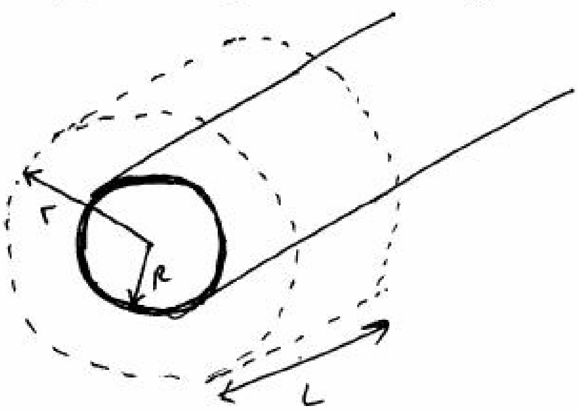
Gauss' Law:

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{s} = q_{enc}$$

$$\varepsilon_0 E(2\pi r L) = 0 \rightarrow E = 0 \quad \text{since } q_{enc} = 0$$

$E = 0$

(b) Find an expression for the magnitude of electric field at a distance r from the tube axis where $r>R$



$$\varepsilon_0 \oint \vec{E}\cdot d\vec{s} = q_{enc}$$

$$\varepsilon_0 \int E\, ds = \int \sigma\, dA$$

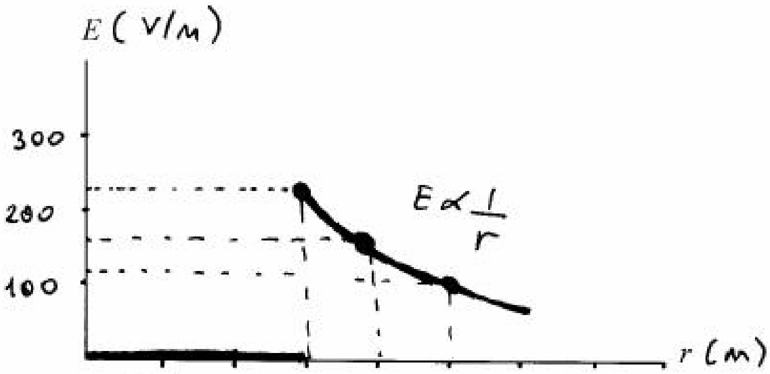
$$\varepsilon_0 E \int ds = \sigma \int dA$$

$$\varepsilon_0 E(2\pi r L) = \sigma(2\pi R L)$$

$$\therefore \quad E = \frac{\sigma R}{\varepsilon_0 r}$$

$E = \sigma R / \varepsilon_0 r$

(c) Plot your results for the range $r = 0$ to $r = 5$ cm, assuming that $\sigma = 2\text{x}10^{-9}$ C/m^2 and $R = 3$ cm



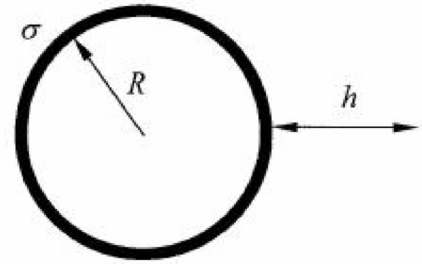
$$E_{in} = 0 \quad (r < R)$$

$$E_{out} = \frac{\sigma R}{\varepsilon_0 r} = \frac{(2\times10^{-9})(0.03)}{(8.85\times10^{-12})\, r} = \frac{6.8}{r}$$

$$E = \begin{cases} 0 & \text{if } r < R \\ 6.8/r & \text{if } r \geq R \end{cases}$$

QUESTION 4 (20 %)

A charged spherical metal shell of radius R = 15 cm has a surface charge distribution of $\sigma = -1.06 \times 10^{-7}$ C/m^2.



(a) What is total charge on the sphere's surface?

$$Q = \text{density} \times \text{surface area}$$
$$= \sigma \times 4\pi R^2$$
$$= \left(-1.06\times10^{-7} \frac{C}{m^2}\right)\left(4\pi\,[0.15\,m]^2\right)$$
$$= -3\times10^{-8}\ C$$

$Q = -3\times10^{-8}$ C

(b) What is the electric potential at the sphere's surface?

$$V = k\frac{Q}{R} = \left(9\times10^{9}\right)\left(\frac{-3\times10^{-8}}{0.15}\right)$$
$$= -1800\ V$$

$V = -1800$ V

(c) At what distance from the sphere's surface has the electric potential increased by 500 V?

potential difference : $\Delta V = V_{R+h} - V_R = 500$ V

$$V_{R+h} = \Delta V + V_R = 500 - 1800 = -1300\,V$$

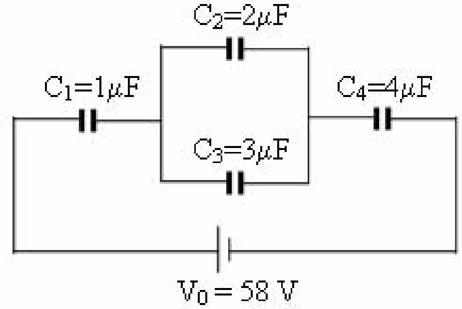
$$V_{R+h} = k\frac{Q}{R+h} \rightarrow h = \frac{kQ}{V_{R+h}} - R$$
$$= \frac{(9\times10^{9})(-3\times10^{-8})}{-1300} - 0.15$$
$$= 0.058\ m$$
$$= 5.8\ cm$$

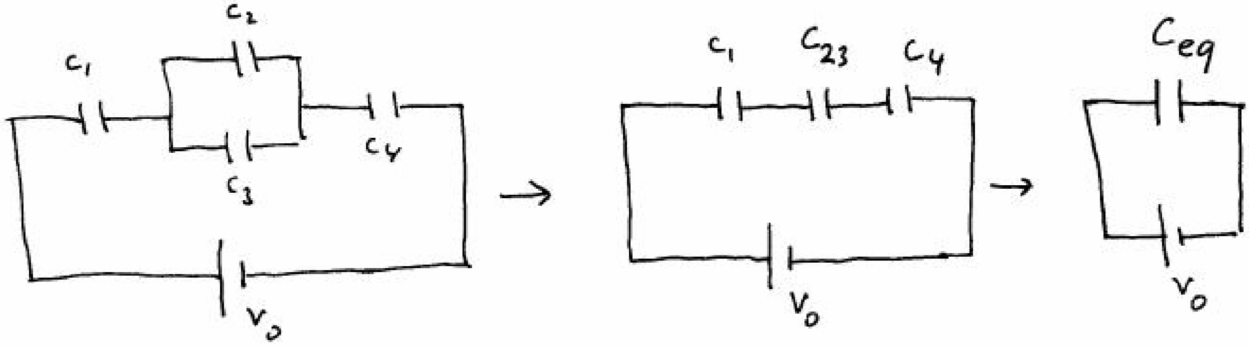
$h = 5.8$ cm

QUESTION 5 (20 %)

Four capacitors are connected together as seen in the figure. Calculate the charges and potential difference on each capacitor.

(Hint: $1\mu F = 10^{-6}$ F)





$$C_{23} = C_2 + C_3 = 2+3 = 5\,\mu F$$

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{1} + \frac{1}{5} + \frac{1}{4}\right)^{-1} = 0.69\,\mu F$$

$$q_1 = q_{23} = q_4 = C_{eq} V_0 = (0.69\,\mu F)(58V) = 40\,\mu C$$

$$V_1 = \frac{q_1}{C_1} = \frac{40\,\mu C}{1\,\mu F} = 40\,V$$

$$V_4 = \frac{q_4}{C_4} = \frac{40\,\mu C}{4\,\mu F} = 10\,V$$

$$V_2 = V_3 = \frac{q_{23}}{C_{23}} = \frac{40\,\mu C}{5\,\mu F} = 8\,V$$

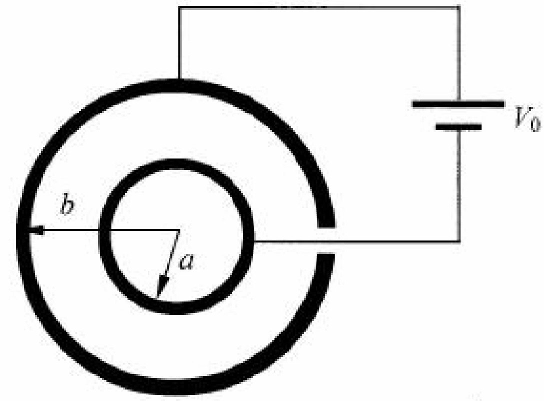
$$q_2 = C_2 V_2 = (2\,\mu F)(8\,V) = 16\,\mu C$$

$$q_3 = C_3 V_3 = (3\,\mu F)(8\,V) = 24\,\mu C$$

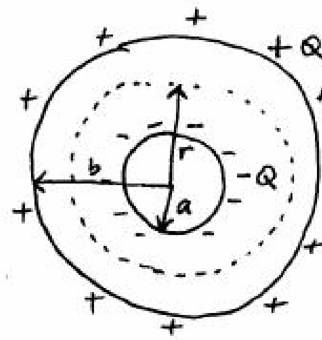
$q_1 =$ 40 μC	$q_2 =$ 16 μC	$q_3 =$ 24 μC	$q_4 =$ 40 μC
$V_1 =$ 40 V	$V_2 =$ 8 V	$V_3 =$ 8 V	$V_4 =$ 10 V

QUESTION 6 (20 %)

A spherical capacitor is formed from two concentric spherical conducting shells separated by air as shown in the figure. Inner sphere has radius $a = 5$ cm and outer has radius $b = 10$ cm. The capacitor is charged to a potential difference $V_0 = 90$ V.



(a) What is the capacitance of the capacitor?



Gauss law: $\varepsilon_0 \oint \vec{E} \cdot d\vec{s} = q_{enc}$

$$\varepsilon_0 E (4\pi r^2) = Q \rightarrow E = \frac{kQ}{r^2}$$

Potential diff. between conductors: $V_{ab} = -\int_a^b \vec{E} \cdot d\vec{l} = -\int_a^b E\,dr = -\int_a^b \frac{kQ}{r^2} dr$

$$= kQ\left(\frac{b-a}{ab}\right)$$

Capacitance:

$$C = \frac{Q}{V_{ab}} = \frac{Q}{kQ\left(\frac{b-a}{ab}\right)} = \frac{ab}{k(b-a)} = \frac{(0.05)(0.1)}{9 \times 10^9 (0.1 - 0.05)} = 1.1 \times 10^{-11} \text{F}$$

$C = 1.1 \times 10^{-11}$F

(b) What charge is collected in outer surface?

$$Q = CV_0 = (1.1 \times 10^{-11})(90) = 1 \times 10^{-9} \text{C} = 1 \text{nC}$$

$Q = 1 \times 10^{-9}$ C

(c) What is the stored energy in the capacitor?

$$U = \frac{1}{2} C V_0^2 = \frac{1}{2}(1.1 \times 10^{-11})(90)^2 = 4.5 \times 10^{-8} \text{J}$$

$U = 4.5 \times 10^{-8}$J

(d) What is the average energy density (stored energy per unit volume) in the capacitor?

$$u = \frac{\text{Energy}}{\text{Volume}} = \frac{U}{\frac{4}{3}\pi(b^3 - a^3)} = \frac{4.5 \times 10^{-8}}{\frac{4}{3}\pi([0.1]^3 - [0.05]^3)}$$

$$= 1.2 \times 10^{-7} \frac{\text{J}}{\text{m}^3}$$

$u = 1.2 \times 10^{-7}$ J/m^3