



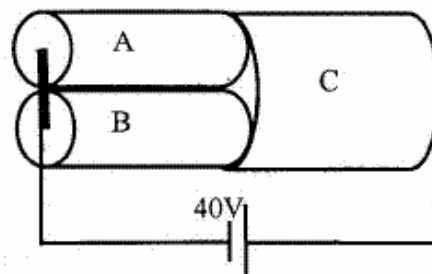
Ques.	Mark
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Total	
Out of	100 %

Name	Surname	Dep.	Signature
SOLUTIONS!!!			

- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes if provided, otherwise your answer will not be considered.
- Useful constants: $g = 9.8 \text{ m/s}^2$, $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$, $e = -1.602 \times 10^{-19} \text{ C}$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$

QUESTION 1 (25 %)

The isolated two conductors A and B having equal length of 10m and radii of 2mm are connected in series to another conductor C having length of 5m and radius of 4mm as seen in figure. The resistivities of the conductors are $1.6 \times 10^{-6} \Omega\text{-m}$, $1.6 \times 10^{-6} \Omega\text{-m}$, and $3.2 \times 10^{-6} \Omega\text{-m}$, respectively. If a potential difference 40 volt is applied between the ends of the composite wire determine:



a) the resistance value of each wire,

$$R_A = \rho_A \frac{L_A}{A_A} \Rightarrow R_A = (1.6 \times 10^{-6}) \times \frac{10}{\pi (2 \times 10^{-3})^2}$$

$$R_A = 1.27 \Omega \text{ and } R_B = 1.27 \text{ since } \rho_A = \rho_B$$

$$R_C = \rho_C \frac{L_C}{A_C} \Rightarrow R_C = (3.2 \times 10^{-6}) \frac{5}{\pi (4 \times 10^{-3})^2}$$

$$R_C = 0.318 \Omega$$

b) the current density in each wire,

$$R_A // R_B \Rightarrow R_{eq} = \frac{R_A R_B}{R_A + R_B} \Rightarrow R_{eq} = 0.635 \Omega \text{ and } R_T = R_{eq} + R_C \Rightarrow R_T = 0.953 \Omega$$

$$V = I R_T \Rightarrow I = \frac{40V}{0.953 \Omega} \Rightarrow I = 41.97 \text{ Amp.}$$

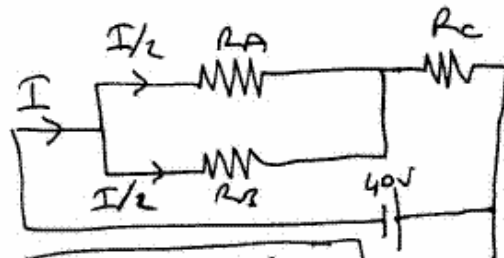
$$J_A = \frac{I/2}{A_A} \Rightarrow J_A = \frac{20.98 \text{ Amp}}{\pi (2 \times 10^{-3})^2} \Rightarrow J_A = 1.67 \times 10^6 \text{ Amp/m}^2 = J_A$$

$$J_C = \frac{I}{A_C} \Rightarrow J_C = \frac{I}{A_C} \Rightarrow J_C = \frac{41.97}{\pi (4 \times 10^{-3})^2} \Rightarrow J_C = 0.835 \times 10^6 \text{ Amp/m}^2$$

c) the potential differences across each wire.

$$V_A = \left(\frac{I}{2}\right) * R_A \Rightarrow V_A = 26.65V = V_A$$

$$V_C = I * R_C \Rightarrow V_C = 13.35V$$

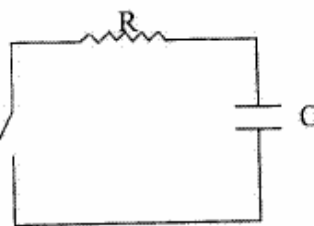


$$R_A = R_B = 1.27 \Omega$$

$$R_C = 0.318 \Omega$$

QUESTION 2 (25 %)

An RC circuit is discharged by closing a switch at time $t=0$. The initial potential difference across the capacitor is 100 V. If the potential difference has decreased to 10 V after 10 sec.



$$V_0 = 100\text{V}, t = 10\text{sec}, V_c(t=10) = 10\text{V}, C = 0.05\mu\text{F}$$

$$t = 12\text{sec} \Rightarrow V_c = ?$$

a) What is the time constant of the circuit?

$$V_c = V_0 e^{-t/\tau} \Rightarrow 10\text{V} = 100\text{V} \times e^{-\frac{10}{\tau}} \Rightarrow \tau = \frac{10}{\ln 10} = \tau$$

$$\tau = RC = 4.34\text{sec} \quad [R = 4.34 / 0.05 \times 10^{-6} \Rightarrow R \approx 8.7 \times 10^7 \Omega]$$

$$\tau = 4.34\text{sec}$$

b) What will the potential difference across the capacitor after $t=12$ sec?

$$V_c = V_0 e^{-t/\tau} \Rightarrow$$

$$V_c(t=12) = 100 e^{-\frac{12}{4.34}}$$

$$V_c(t=12) = \underline{6.31\text{V}}$$

$$V_c = 6.31\text{V}$$

c) What will the amount of charge be on each plate of the capacitor after $t=12$ sec?

$$q_c = q_0 e^{-t/\tau} \Rightarrow q_c(t=12) = (V_0 C) e^{-t/\tau}$$

$$q_c(t=12\text{sec}) = (0.05 \times 10^{-6})(100) e^{-\frac{12}{4.34}} \Rightarrow q_c = \underline{3.15 \times 10^{-7}\text{C}}$$

$$Q_c = 3.15 \times 10^{-7}\text{C}$$

d) What is the current through the resistor after $t=12$ sec? if $C=0.05\mu\text{F}$.

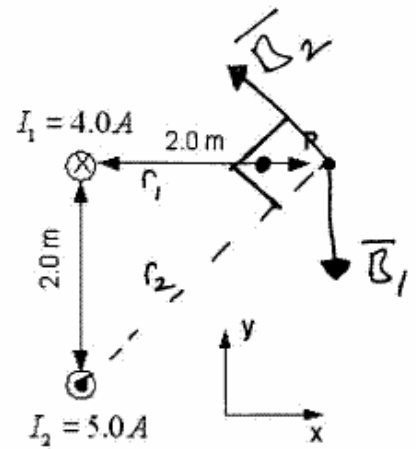
$$I_R = I_0 e^{-t/\tau} \Rightarrow I_R(t=12) = \left(\frac{V_0}{R}\right) e^{-t/\tau}$$

$$I_R = \frac{100}{8.7 \times 10^7} \times e^{-12/4.34} \Rightarrow I_R = \underline{7.25 \times 10^{-9}\text{Amp}}$$

$$I_R = 7.25 \times 10^{-9}\text{A}$$

QUESTION 3 (25 %)

Two wires carrying currents I_1 and I_2 are 2.0 m apart each other. A point P is located as shown in Figure.



- a) Find the magnitude and direction of the resultant magnetic field at point P .

$$|\vec{B}_1| = \frac{\mu_0 I_1}{2\pi r_1} \Rightarrow |\vec{B}_1| = 4.0 \times 10^{-7} \text{ T}$$

$$\text{and } \vec{B}_1 = 4.0 \times 10^{-7} \text{ T } (-\hat{y})$$

$$|\vec{B}_2| = \frac{\mu_0 I_2}{2\pi r_2} \Rightarrow |\vec{B}_2| = 2.54 \times 10^{-7} \text{ T} \text{ and } \vec{B}_2 \text{ has } x \text{ and } y \text{ components:}$$

$$|\vec{B}_{2x}| = |\vec{B}_2| \cos(45^\circ) \Rightarrow \vec{B}_{2x} = 2.51 \times 10^{-7} \text{ T } (-\hat{x}) \text{ and } \vec{B}_{2y} = \vec{B}_2 \sin(45^\circ) (\hat{y})$$

$$\vec{B}_{2y} = 2.48 \times 10^{-7} \text{ T } (-\hat{y}), \text{ So that the } \vec{B} \text{ has 2 components:}$$

$$\vec{B}_x = 2.51 \times 10^{-7} \text{ T } (-\hat{x})$$

$$\vec{B}_y = \vec{B}_1 + \vec{B}_{2y} \Rightarrow \vec{B}_y = 1.52 \times 10^{-7} \text{ T } (-\hat{y}) \Rightarrow |\vec{B}| = 2.9 \times 10^{-7} \text{ T}$$

$$\text{and } \theta = \tan^{-1}\left(\frac{|\vec{B}_x|}{|\vec{B}_y|}\right) \Rightarrow \theta = 31^\circ \text{ below } (-x) \text{ direction.}$$

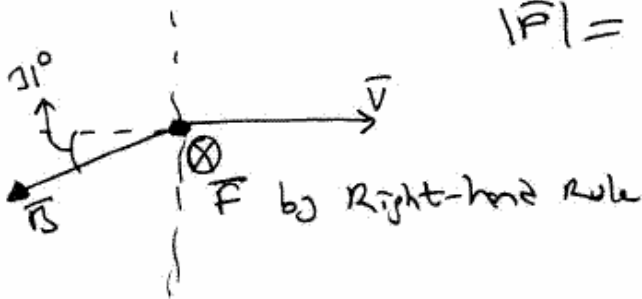
$$|\vec{B}| = 2.9 \times 10^{-7} \text{ T in the } (-x/y) \text{-direction}$$

- b) At some instant in time, a proton is at point P moving with a speed of $5 \times 10^6 \text{ m/s}$ in the x -direction. What is the magnitude and direction of the magnetic force on the proton.

$$\vec{F} = q \vec{v} \times \vec{B} \Rightarrow |\vec{F}| = qvB \sin(\theta)$$

$$|\vec{F}| = (+1.6 \times 10^{-19})(5 \times 10^6)(2.9 \times 10^{-7}) \sin(149^\circ)$$

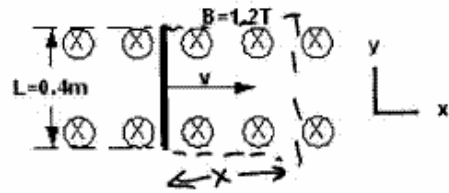
$$|\vec{F}| = 1.2 \times 10^{-19} \text{ N, in the direction of 'into the paper' } \otimes$$



$$|\vec{F}| = 1.2 \times 10^{-19} \text{ N in the } (\otimes) \text{-direction}$$

QUESTION 4 (25 %)

An iron rod of length 0.40 m moves with a velocity v in a magnetic field, of magnitude $B=1.2\text{ T}$ that is perpendicular to the direction of motion of the rod, as shown in Figure. The EMF (Electro-motive force) induced in the moving rod is found to be 2.40 V



a) What is the speed of the rod?

$$\mathcal{E} = - \frac{d\Phi_B}{dt}, \quad \Phi_B = \mathbf{B} \cdot \mathbf{S} \quad \text{where } \mathbf{B} \perp \mathbf{S} \text{ then } S = L \cdot x$$

$$\text{and } \mathcal{E} = - \frac{d}{dt} (B \cdot L \cdot x) \Rightarrow |\mathcal{E}| = |BL \frac{dx}{dt}| \Rightarrow |\mathcal{E}| = BL|v| \text{ then}$$

$$|v| = \frac{\mathcal{E}}{BL} \Rightarrow |v| = \frac{2.4\text{V}}{1.2 \cdot 0.4} \Rightarrow |v| = 5\text{m/s}$$

$|v| = 5\text{m/s.}$

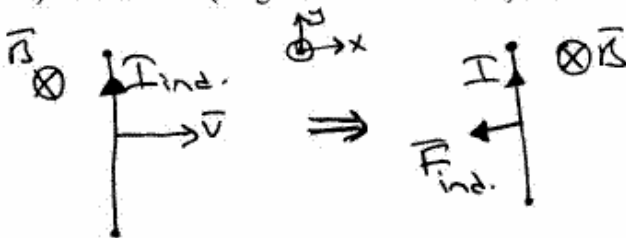
b) If the total resistance of the rod is assumed to be 1.2 ohms , what is the induced current on the rod?

Since $V = IR$ (ohm's Law); then one writes

$$|\mathcal{E}| = I \cdot R \Rightarrow I = \frac{|\mathcal{E}|}{R} \Rightarrow I_{\text{induced}} = \frac{2.4\text{V}}{1.2\Omega} \Rightarrow I_{\text{ind}} = 2\text{Amp}$$

$I = 2.0\text{ A}$

c) What force (magnitude and direction) does the field exert on the rod as a result of this current?



$$\text{So, } \mathbf{F} = I \mathbf{L} \times \mathbf{B}$$

$$|\mathbf{F}| = I L B \sin(90^\circ)$$

$$|\mathbf{F}| = I L B \text{ then}$$

$$|\mathbf{F}| = 2 \cdot 0.4 \cdot 1.2, \text{ and}$$

$$|\mathbf{F}| = 0.960\text{N}$$

$|\mathbf{F}| = 0.960\text{N in the } (-\hat{x})\text{-direction}$