UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 106 General Physics II
Second Midterm Exam Questions

Date: 25/05/2006 Time: 100 min.

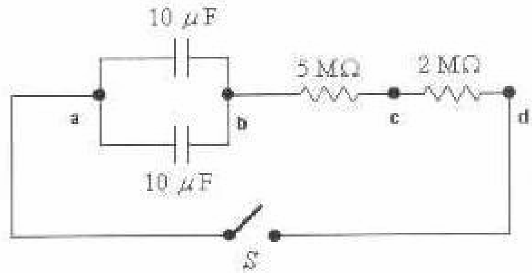
Ques.	Mark
1	
2	
3	
4	
5	
6	xxxxx
Total	
Out of	100 %

Name	Surname	Dep.	Signature
Solutions !			

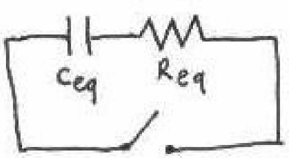
- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Useful constants: $k = 1/4\pi\epsilon_0 = 9\times10^9$ N·m²/C², $e = 1.6\times10^{-19}$ C, $\mu_0 = 4\pi\times10^{-7}$ T·m/A

QUESTION 1 (20 %)

Two capacitors and two resistors are connected to form an RC circuit as shown in Figure. The total initial charge on capacitor system is $Q_0 = 200\ \mu$C. The switch S is closed at a time $t = 0$ s and then capacitors start to discharge owing to the resistors in the circuit.
(Note that: 1 μF = 10^{-6} F and 1 MΩ = 10^6 Ω)



(a) Find the time constant of the circuit.



$C_{eq} = (10+10)\mu F = 20\mu F$ and $R_{eq} = (5+2) M\Omega = 7 M\Omega$

$\tau = R_{eq} C_{eq} = (20\times10^{-6})(7\times10^{6})$
$= 140\ s$

$\boxed{\tau = 140\ s}$

(b) How long does it take to drop the total charge on the capacitor system to Q = 100 μC?

$Q = Q_0 e^{-t/\tau}$

$e^{-t/\tau} = \dfrac{Q}{Q_0}$

$t = -\tau \ln(Q/Q_0)$
$= -140 \ln(100/200)$
$= 97 s$

$\boxed{t = 97\ s}$

(c) Find the current in the circuit when the charge on the capacitor system is Q = 100 μC.

$i = \dfrac{dQ}{dt} = \dfrac{d}{dt}\left[Q_0 e^{-t/\tau}\right] = -\dfrac{Q_0}{\tau} e^{-t/\tau}$

$= \dfrac{200\times10^{-6}}{140} e^{-97/140}$

$i = 7.1\times10^{-7} A$

$\boxed{i = 7.1\times10^{-7} A}$

(d) Find the potential energy stored in each capacitor at t = 2 minutes.

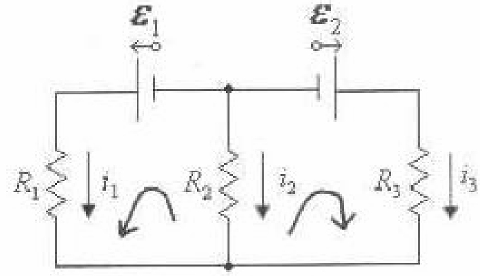
at $t = 2\ min = 120\ s$
total charge:
$Q = 200\, e^{-120/140} = 85\ \mu C$
for each capacitor:
$Q' = Q/2 = 42.5\ \mu C$

$U = U_1 = U_2 = \dfrac{Q'^2}{2C}$
$= \dfrac{(42.5\times10^{-6})^2}{(2)(10\times10^{-6})}$
$= 9\times10^{-6}\ J$

$\boxed{U = 9.0\times10^{-6}\ J}$

QUESTION 2 (20 %)

Two batteries with $\epsilon_1 = 3$ V and $\epsilon_2 = 5$ V are connected with three resistors R_1=10 Ω, R_2 = 20 Ω and R_3 = 30 Ω as shown in Figure.



Using Kirchhoff's laws, find the currents i_1, i_2 and i_3 passing through the resistors.

$$\varepsilon_1 - i_1 R_1 + i_2 R_2 = 0$$
$$\varepsilon_2 - i_3 R_3 + i_2 R_2 = 0$$
$$i_1 + i_2 + i_3 = 0$$

$$\left.\begin{array}{l} 3 - 10 i_1 + 20 i_2 = 0 \\ 5 - 30 i_3 + 20 i_2 = 0 \\ i_1 + i_2 + i_3 = 0 \end{array}\right\}$$

Solving:

$i_1 = 0.045$ A

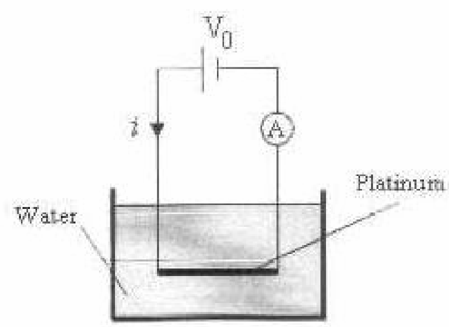
$i_2 = -0.127$ A

$i_3 = 0.082$ A

$i_1 =$	0.045 A
$i_2 =$	− 0.127 A
$i_3 =$	0.082 A

QUESTION 3 (20 %)

A resistance thermometer made from Platinum has a resistance 50 Ω at 20 °C. When the thermometer is immersed in a water container as shown in Figure, the ammeter in the circuit indicates 0.15 A. If battery supplies V_0 = 9 V, what is the temperature of the water?



Temperature coefficient of the Platinum is 4.0×10^{-3} 1/°C.

$T_0 = 20$ °c $\rightarrow$ $R_0 = 50\ \Omega$

$$\frac{V_0}{i} = R = R_0(1 + \alpha [T - T_0])$$

Solving for T:

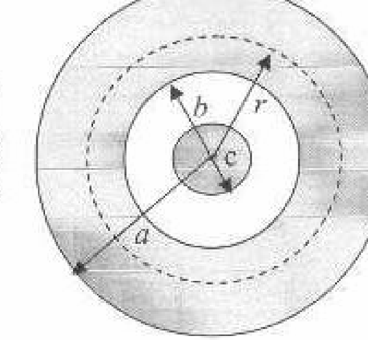
$$T = \frac{1}{\alpha}\left(\frac{V_0/i}{R_0} - 1\right) + T_0$$
$$= \frac{1}{4\times10^{-3}}\left(\frac{9/0.15}{50} - 1\right) + 20$$

T = 70 °c

$T =$	70 °C

QUESTION 4 (20 %)

Figure shows a cross section of a long conductor of a type called coaxial cable. Its dimensions are labeled in the figure and given as a = 2 cm, b = 1.8 cm and c = 0.4 cm. There are equal but opposite stable currents of I=100 A in the two conductors. Using Ampere's law, derive expressions for B(r) and calculate its value in the ranges:



(a) r = 0.2 cm (r<c)

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc} \rightarrow B(2\pi r) = \mu_0 \frac{\pi r^2}{\pi c^2} I \rightarrow B = \frac{\mu_0 I r}{2\pi c^2}$$

$$B(0.2\text{ cm}) = \frac{(4\pi\times10^{-7})(100)(0.2\times10^{-2})}{(2\pi)(0.4\times10^{-2})^2}$$

$$= 2.5\times10^{-3}\ \text{T}$$

$B(r)$ =	$\mu_0 I r / 2\pi c^2$
B(0.2 cm) =	2.5×10^{-3} T

(b) r =1.4 cm (c<r<b),

$$B(2\pi r) = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

$$B(1.4\text{ cm}) = \frac{(4\pi\times10^{-7})(100)}{(2\pi)(1.4\times10^{-2})} = 1.4\times10^{-3}\ \text{T}$$

$B(r)$ =	$\mu_0 I / 2\pi r$
B(1.4 cm) =	1.4×10^{-3} T

(c) r = 1.9 cm (b<r<a)

$$B(2\pi r) = \mu_0\left(I - \frac{\pi r^2 - \pi b^2}{\pi a^2 - \pi b^2} I\right)$$

$$B = \frac{\mu_0 I}{2\pi r}\left(1 - \frac{r^2 - b^2}{a^2 - b^2}\right) \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right)$$

$$B(1.9\text{ cm}) = \frac{(4\pi\times10^{-7})(100)}{(2\pi)(1.9\times10^{-2})}\left(\frac{2^2 - 1.9^2}{2^2 - 1.8^2}\right)$$

$$= 5.4\times10^{-4}\ \text{T}$$

$B(r)$ =	$\frac{\mu_0 I}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right)$
B(1.9 cm) =	5.4×10^{-4} T

(d) r = 6 cm (r>a)

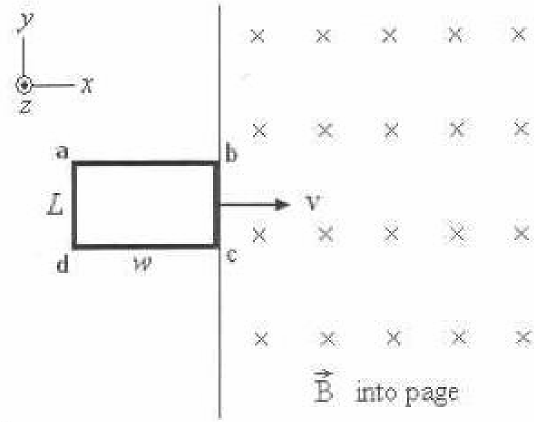
$$B(2\pi r) = \mu_0 i_{enc} \rightarrow B = 0$$

$$\text{since } i_{enc} = I - I = 0$$

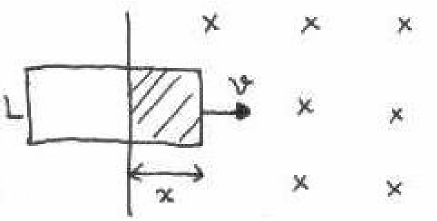
$B(r)$ =	0
B(6.0 cm) =	0

QUESTION 5 (20 %)

A thin wire of resistance $R = 1\ \Omega$ is bent to form a rectangular loop abcd of sides are $L = 10$ cm and $w = 20$ cm. This loop travels at constant velocity v = 2.5 cm/s trough a region containing a uniform magnetic filed of $B = 0.2$ T such that its normal is perpendicular to the field direction. Assume that the position of the loop at time $t = 0$ s is $x = y = z = 0$ as shown in Figure.



(a) Find the magnetic flux enclosed by the loop at time $t = 2$ s.



$$\Phi_B = \int \vec{B}\cdot d\vec{A} = BA$$

$$\Phi_B = B(L\cdot x) = BLvt = (0.2)(0.1)(0.025)(2)$$

$$\Phi_B = 1.0\times 10^{-3}\ \mathrm{Wb/m^2}$$

$\boxed{\Phi_B = 1.0\times 10^{-3}\ \frac{\mathrm{Wb}}{\mathrm{m^2}}}$

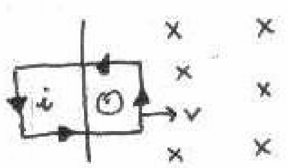
(b) Find the induced electromotive force (emf) at time $t = 2$ s.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[BLvt\right] = -BLv = -(0.2)(0.1)(0.025)$$

$$\mathcal{E} = -5.0\times 10^{-4}\ V$$

$\boxed{\varepsilon = -5.0\times 10^{-4}\ V}$

(c) Find the magnitude of induced current and its direction in the wire at time $t = 2$ s.

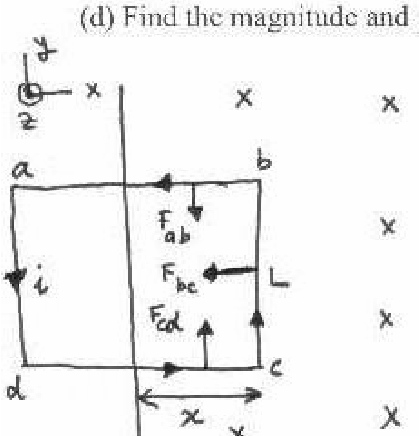


Direction of the current can be found from right hand rule using Lenz's Law.

$$i = \frac{|\mathcal{E}|}{R} = \frac{5\times 10^{-4}\ V}{1\ \Omega} = 5\times 10^{-4}\ A$$

$\boxed{i = 5.0\times 10^{-4}\ A}$

(d) Find the magnitude and show the direction of forces acting on each segment of the loop at $t = 2$s.



$x = (2.5\ \mathrm{cm/s})(2s) = 5.0$ cm

$$\vec{F}_{ab} = ix(-\hat{i}) \times B(-\hat{k}) = -iBx\,\hat{j} = -(5\times 10^{-4})(0.2)(0.05)\,\hat{j} = -\hat{j}\,5.0\times 10^{-6}\ N$$

$$\vec{F}_{bc} = iL(\hat{j}) \times B(-\hat{k}) = -iBL\,\hat{i} = -(5\times 10^{-4})(0.2)(0.1)\,\hat{i} = -\hat{i}\ 10\times 10^{-6}\ N$$

$$\vec{F}_{dc} = -\vec{F}_{ab} = +\hat{j}\ 5\times 10^{-6}\ N$$

$\vec{F}_{da} = 0$ since $B = 0$ outside

$F_{ab} = 5\times 10^{-6}\ N$	$F_{bc} = 10\times 10^{-6}\ N$	$F_{cd} = 5\times 10^{-6}\ N$	$F_{da} = 0$