

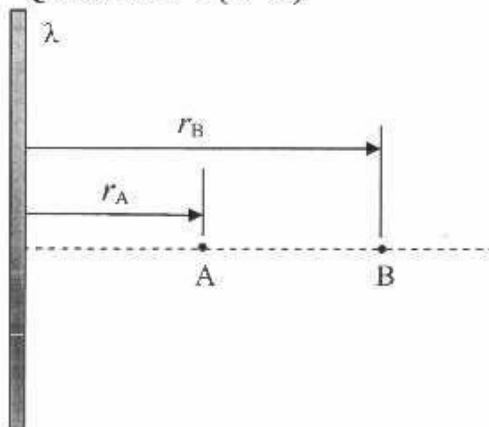


Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
Out of	100 %

Name	Surname	Dep.	Signature
Solutions		EP	

- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Useful constants: $k = 1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$, $\sin 30 = 1/2$, $\cos 30 = \sqrt{3}/2$, $e = 1.6 \times 10^{-19} \text{ C}$

QUESTION 1 (17 %)

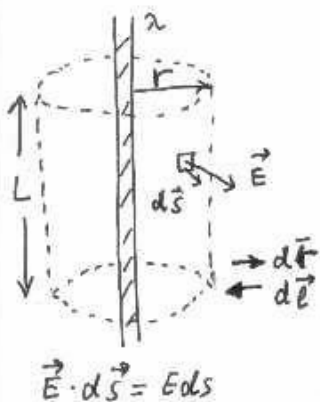


A very long insulating rod carries a constant linear charge density $\lambda = +0.5 \times 10^{-9} \text{ C/m}$.

Find the work done in moving a point charge $q = +15 \times 10^{-9} \text{ C}$ from point B to point A?

Assume that $r_A = 60 \text{ cm}$ and $r_B = 120 \text{ cm}$.

Electric field:



$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = q_{enc}$$

$$\epsilon_0 (E 2\pi r L) = qL$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$= \frac{0.5 \times 10^{-9}}{(2\pi)(8.85 \times 10^{-12})r}$$

$$E = \frac{9}{r} \frac{\text{V}}{\text{m}}$$

here r is in meter.

Potential difference:

$$V_A - V_B = - \int_B^A \vec{E} \cdot d\vec{\ell} = - \int_B^A E dl \cos 180^\circ$$

Since $dl = -dr$

$$V_A - V_B = - \int_{r_B}^{r_A} E dr$$

$$= - \int_{1.2}^{0.6} \frac{9}{r} dr$$

$$= 9 \ln 2 \approx \underline{6.24 \text{ V}}$$

Work done:

$$W_{BA} = (V_A - V_B) q$$

$$= (6.24)(15 \times 10^{-9} \text{ C})$$

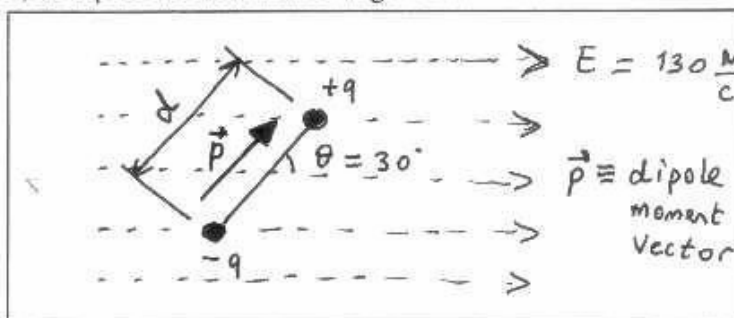
$$= +9.36 \times 10^{-9} \text{ J}$$

$$W_{BA} = 9.36 \times 10^{-9} \text{ J}$$

QUESTION 2 (17%)

A molecule having two atoms has a dipole moment of $6 \times 10^{-30} \text{ C}\cdot\text{m}$. The distance between the atoms is known as $3.75 \times 10^{-11} \text{ m}$. When the molecule is placed in an external uniform electric field of magnitude 130 N/C , its dipole moment makes angle of 30° with that field.

- (a) Draw the system (inside the box given right) for these conditions and show the direction of the dipole moment vector explicitly.



- (b) Calculate the amount of charge that generates the dipole moment

$$q = \frac{p}{d} = \frac{6 \times 10^{-30} \text{ C}\cdot\text{m}}{3.75 \times 10^{-11} \text{ m}} = 1.6 \times 10^{-19} \text{ C} \equiv e$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

- (c) Calculate the magnitude of torque on the dipole.

$$\begin{aligned} |\vec{\tau}| &= |\vec{p} \times \vec{E}| \\ &= p E \sin \theta = (6 \times 10^{-30} \text{ C}\cdot\text{m})(130 \frac{\text{N}}{\text{C}}) \sin 30^\circ \\ &= 3.9 \times 10^{-28} \text{ N}\cdot\text{m} \end{aligned}$$

$$\tau = 3.9 \times 10^{-28} \text{ N}\cdot\text{m}$$

- (d) Calculate the potential energy of the molecule in that electric field.

$$\begin{aligned} U &= -\vec{p} \cdot \vec{E} = -p E \cos \theta \\ &= -(6 \times 10^{-30} \text{ C}\cdot\text{m})(130 \frac{\text{N}}{\text{C}}) \cos 30^\circ \\ &= -6.8 \times 10^{-28} \text{ J} \end{aligned}$$

$$U = -6.8 \times 10^{-28} \text{ J}$$

QUESTION 3 (17%)

The electrostatic force between two like ions are separated by a distance of $5.0 \times 10^{-10} \text{ m}$ is $3.7 \times 10^{-9} \text{ N}$.

- (a) What is the charge on each ion?

$$\begin{aligned} F &= k \frac{q^2}{r^2} \rightarrow q = \sqrt{\frac{F}{k}} r \\ &= \sqrt{\frac{3.7 \times 10^{-9}}{9 \times 10^9}} (5 \times 10^{-10}) \\ &= 3.2 \times 10^{-19} \text{ C} \end{aligned}$$



$$q = 3.2 \times 10^{-19} \text{ C}$$

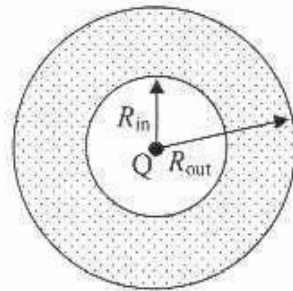
- (b) How many electrons are missing from each ion?

$$n = \frac{q}{e} = \frac{3.2 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2$$

$$n = 2$$

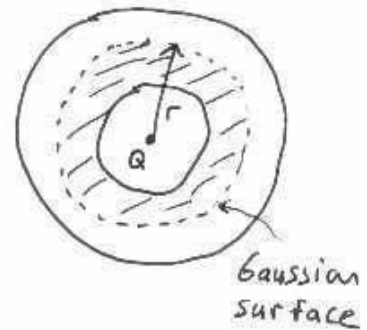
QUESTION 4 (17 %)

Consider a point charge $Q = +4.0 \text{ nC}$ placed at the center of a non-conducting shell which has the same charge Q distributed uniformly in the sphere. The inner and outer radius of the shell are $R_{in} = 2 \text{ cm}$ and $R_{out} = 4 \text{ cm}$, respectively.



(a) What is the magnitude of electric field at a distance r from the center of the shell if $r = 3 \text{ cm}$?

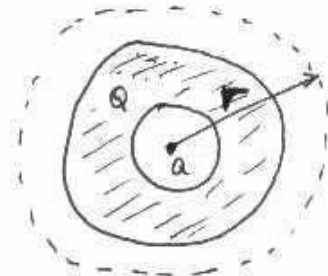
$$\begin{aligned} \epsilon_0 \oint \vec{E} \cdot d\vec{S} &= q_{enc} \\ \epsilon_0 E (4\pi r^2) &= Q + \frac{\frac{4\pi}{3} (r^3 - R_{in}^3)}{\frac{4\pi}{3} (R_{out}^3 - R_{in}^3)} Q \\ E &= \frac{Q}{4\pi\epsilon_0 r^2} \left[1 + \frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3} \right] \\ &= \frac{4 \times 10^{-9}}{(4\pi)(8.85 \times 10^{-12})(3 \times 10^{-2})^2} \left[1 + \frac{3^3 - 2^3}{4^3 - 2^3} \right] = 1.2 \times 10^5 \text{ N/C} \end{aligned}$$



$$E = 1.2 \times 10^5 \text{ N/C}$$

(b) What is the magnitude of electric field at a distance r from the center of the shell if $r = 8 \text{ cm}$?

$$\begin{aligned} \epsilon_0 \oint \vec{E} \cdot d\vec{S} &= q_{enc} \\ \epsilon_0 E (4\pi r^2) &= Q + Q \\ E &= \frac{2Q}{4\pi\epsilon_0 r^2} = \frac{(2)(9 \times 10^9)(4 \times 10^{-9})}{(8 \times 10^{-2})^2} = 1.1 \times 10^4 \text{ N/C} \end{aligned}$$



$$E = 1.1 \times 10^4 \text{ N/C}$$

QUESTION 5 (15 %)

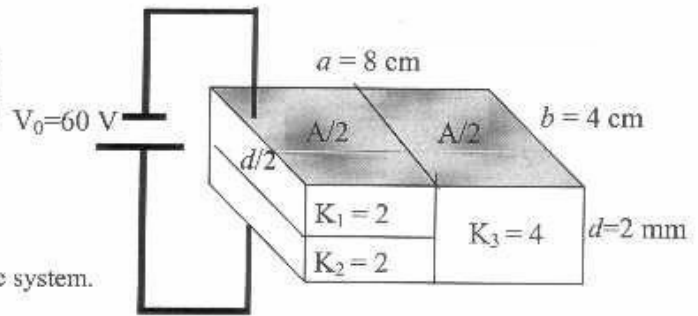
The electric potential in a region is given by $V(x, y, z) = x^2 - 4y + 3yz^2$ (volt) where x, y and z are in meters. Find the direction of electric field in unit vector notation at the point $P(1.0 \text{ m}, 2.0 \text{ m}, 1.0 \text{ m})$.

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -(2x) = -2x \\ E_y &= -\frac{\partial V}{\partial y} = -(-4 + 3z^2) = 4 - 3z^2 \\ E_z &= -\frac{\partial V}{\partial z} = -(6yz) = -6yz \\ \vec{E}(x, y, z) &= -2x \hat{i} + (4 - 3z^2) \hat{j} - 6yz \hat{k} \\ \vec{E}(1, 2, 1) &= -(2)(1) \hat{i} + (4 - 3(1)^2) \hat{j} - (6)(2)(1) \hat{k} \text{ (N/C)} \\ \vec{E} &= -2 \hat{i} + \hat{j} - 12 \hat{k} \text{ (N/C)} \end{aligned}$$

$$\vec{E} = -2 \hat{i} + \hat{j} - 12 \hat{k} \frac{\text{N}}{\text{C}}$$

QUESTION 6 (17%)

A rectangular parallel plate capacitor having sides a and b and separation d is filled with three dielectric materials as seen in the Figure.



(a) Find the equivalent capacitance value of the system.

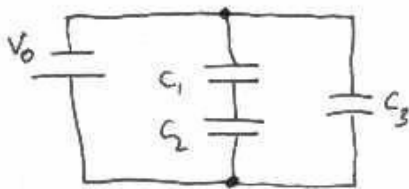


Plate area: $A = ab = (0.08)(0.04) = 3.2 \times 10^{-3} \text{ m}^2$

$$C_1 = \frac{\epsilon_0 K_1 (A/2)}{d/2} = \frac{\epsilon_0 K_1 A}{d} = \frac{(8.85 \times 10^{-12})(2)(3.2 \times 10^{-3})}{2 \times 10^{-3}} = 2.8 \times 10^{-11} \text{ F}$$

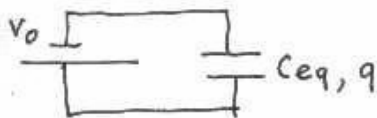
$$C_2 = \frac{\epsilon_0 K_2 (A/2)}{d/2} = \frac{\epsilon_0 K_2 A}{d} = \frac{(8.85 \times 10^{-12})(2)(3.2 \times 10^{-3})}{2 \times 10^{-3}} = 2.8 \times 10^{-11} \text{ F}$$

$$C_3 = \frac{\epsilon_0 K_3 (A/2)}{d} = \frac{\epsilon_0 K_3 A}{2d} = \frac{(8.85 \times 10^{-12})(4)(3.2 \times 10^{-3})}{(2)(2 \times 10^{-3})} = 2.8 \times 10^{-11} \text{ F}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{C_1}{2} + C_3 = \frac{3C_1}{2} = \frac{3}{2} (2.8 \times 10^{-11} \text{ F}) = 4.2 \times 10^{-11} \text{ F}$$

$C_{eq} = 4.2 \times 10^{-11} \text{ F}$

(b) Find the charge on each capacitor (dielectric).



$$q = C_{eq} V_0 = (4.2 \times 10^{-11})(60) = 2.5 \times 10^{-9} \text{ C}$$

$$q_2 = C_3 V_0 = (2.8 \times 10^{-11})(60) = 1.7 \times 10^{-9} \text{ C}$$

$$q_{12} = q - q_3 = (2.5 - 1.7) \times 10^{-9} = 0.8 \times 10^{-9} \text{ C}$$

$$q_1 = q_2 = q_{12}$$

$$q = q_{12} + q_3$$

$q_1 = 0.8 \times 10^{-9} \text{ C}$
$q_2 = 1.7 \times 10^{-9} \text{ C}$
$q_3 = 1.7 \times 10^{-9} \text{ C}$