

Chapter 5: Newton's Laws of Motion (Particle Dynamics I)

Introduction

Having learned how to describe motion, we can now turn to the more fundamental question of what causes motion. An object is set into motion when it is pushed or pulled or subjected to a *force*. The force can be described by the effect that accelerates or decelerates the object or make changes in its position or in its shape. Such effects are described accurately by general laws of motion, were first stated fully by Sir Isaac Newton (1642-1727).

In his *Principia*, Newton reduced the basic principles of mechanics to three laws:

“Every body continues in its state of rest, or uniform motion in a straight line, unless compelled to change that state by forces impressed upon it. The change of motion of an object is proportional to the force impressed upon it, and is made in the direction of the straight line in which the force is impressed. To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.”

These laws are known as Newton's first law of motion, Newton's second law of motion, and Newton's third law of motion, respectively. In this lecture, we shall examine each of these laws in detail, and then give some simple illustrations of their use.

Newton's first law of motion (*Law of Inertia*)

After Aristotle, Galileo has spent much time for observing motion of a body. He has recognized the motion of a body occurs under some forces that describe the motion and there is any reason somehow the motion ends. So, Newton's first law was actually discovered by Galileo and perfected by Descartes (who added the crucial proviso “in a straight line”). This law states that if the motion of a given body is not disturbed by external influences then the body moves with *constant* velocity. In other words:

“If an object is at rest then it will remain at rest or if it is moving along a straight line with uniform speed the it will continue to keep moving unless an external force is applied on it to change its existing state.” (Newton's First Law of motion is also known as **law of inertia**).

This law defines us the force while the second law provide us the means how to measure the force. Mathematically, the displacement \vec{r} of the body as a function of time t can be written

$$\vec{r} = \vec{r}_0 + \vec{v} \times t \quad (5.1)$$

where \vec{r} and \vec{v} are *constant* vectors. As illustrated in Figure 1, the body's trajectory is a *straight-line* which passes through point \vec{r}_0 at time $t=0$ and runs parallel to \vec{v} . The special case is in which $\vec{v} = 0$ the body simply remains at rest.

Nowadays, Newton's first law strikes us as almost a statement of the obvious. However, in Galileo's time this was far from being the case. From the time of the ancient Greeks, philosophers--observing that objects set into motion on the Earth's surface eventually come to rest--had concluded that the natural state of motion of objects was that they should remain at rest. Hence, they reasoned, any object that moves does so under the influence of an external influence, or *force*, exerted on it by some other object. It took the genius of Galileo to realize that an object set into motion on the Earth's surface eventually comes to rest under the influence of frictional forces, and that if these forces could somehow be abstracted from the motion then it would continue forever.

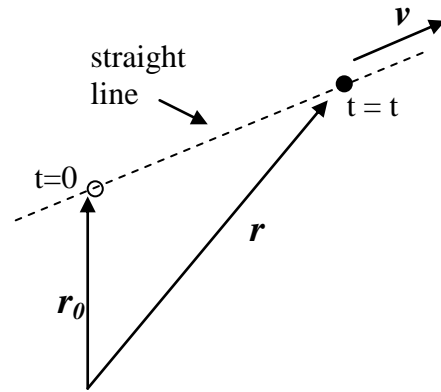


Figure 1: Straight line motion

Newton's second law of motion (*Fundamental Law of Dynamics*)

Newton's first law states that when no net force acts on an object, it stays at rest or in motion with a constant velocity. The second law tells us what happens when this force is not zero. Newton used the word "*motion*" to mean what we nowadays call *momentum*. It is necessary to take into account since the change in the momentum in time is the net force applied on the object.

The second law tells us that

"The rate of change of momentum of a body is proportional to the applied force and the change takes in the direction in which the force acts. There is a proportionality constant called mass, the proportion of the force to the acceleration, that is always constant for a given object."

To obtain the mathematical representations, let the momentum \vec{P} of a body be simply defined as the product of its mass m and its velocity \vec{v} : *i.e.*,

$$\vec{P} = m\vec{v} \quad (5.2)$$

Since the change in the momentum in time Δt causes a force, then

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t} \quad (5.3)$$

where the \vec{F} vector represents the net influence, or force, exerted on the object, whose motion is under investigation, by other objects. Then the momentum change is given by Equation (5.1)

$$\vec{P} = m\vec{v} \Rightarrow \Delta \vec{P} = m\Delta \vec{v} \quad (5.4)$$

inserting this result into the Equation (5.3), we get

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t} = \frac{m\Delta \vec{v}}{\Delta t} \Rightarrow \vec{F} = m \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \vec{F} = m\vec{a} \text{ that means } \sum \vec{F} = m\vec{a} \quad (5.5)$$

As seen in the above; an object with *constant* mass, the above law reduces to its more conventional form. In other words, the net force exerted on a given object by other objects equals the product of that object's mass and its acceleration. Of course, this law is entirely devoid of content unless we have some independent means of quantifying the forces exerted between different objects. In the Equation (5.5), assume that values have values of 1. Then the Equation (5.5) states that $1\text{kg}\cdot\text{m}/\text{s}^2$ Force to a 1kg of mass gives $1\text{m}/\text{s}^2$ acceleration to this mass. So, the unit of Force is $\text{kg}\cdot\text{m}/\text{s}^2$ that is called *1N (1 Newton)*.

Newton's third law of motion (*The Law of Reaction*)

Suppose that there are just two objects in the Universe: You and the Earth! And you are walking on a street. At each step, your feet exert a force to the ground to move forward. Is it true? Maybe the ground exerts some force in the opposite direction to you to slide under your feet!☺

Suppose, now, that there are many objects in the Universe (as is, indeed, the case). According to Newton's third law, if object *b* exerts a force f_{ab} on object *a* then object *a* must exert an equal and opposite force $f_{ba} = -f_{ab}$ on object *b*, as seen Figure 2. That means *b* applies an **action** on *a*, then as a result of this, *a* applies a **reaction** on *b*. It follows that all of the forces acting in the Universe can ultimately be grouped into equal and opposite **action-reaction** pairs. Note, incidentally, that an action and its associated reaction always act on **different bodies**.

The third law states that

“To every action there is always an equal and opposite reaction.”

Why do we need Newton's third law?

Actually, it is almost a matter of common sense. Suppose that bodies *a* and *b* constitute an

isolated system. If $f_{ba} = -f_{ab}$ then this system exerts a *non-zero net force* $f = f_{ab} + f_{ba}$ on itself, without the aid of any external agency. It will, therefore, accelerate forever under its own steam.



Figure 2: Newton's Third Law (Action-Reaction)

We know, from experience, that this sort of behaviour does not occur in real life. For instance, I cannot grab hold of my shoelaces and, thereby, pick myself up off the ground. In other words, I cannot self-generate a force that will spontaneously lift me into the air: I need to exert forces on other objects around me in order to achieve this. Thus, Newton's third law essentially acts as a guarantee against the absurdity of self-generated forces.

Mass and Weight

In fact, the terms *mass* and *weight* are often confused with one another. However, in physics their meanings are quite distinct.

An object's mass is a measure of its *inertia*: This means its reluctance to deviate from uniform straight-line motion under the influence of external forces. According to Newton's second law, Equation (5.5), if two objects of differing masses are acted upon by forces of the same magnitude, then the resultant acceleration of the larger mass is less than that of the smaller mass. In other words, it is more difficult to force the larger mass to deviate from its preferred state of uniform motion in a straight line. Incidentally, the mass of a body is an intrinsic property of that body, and, therefore, does not change if the body is moved to a different place.

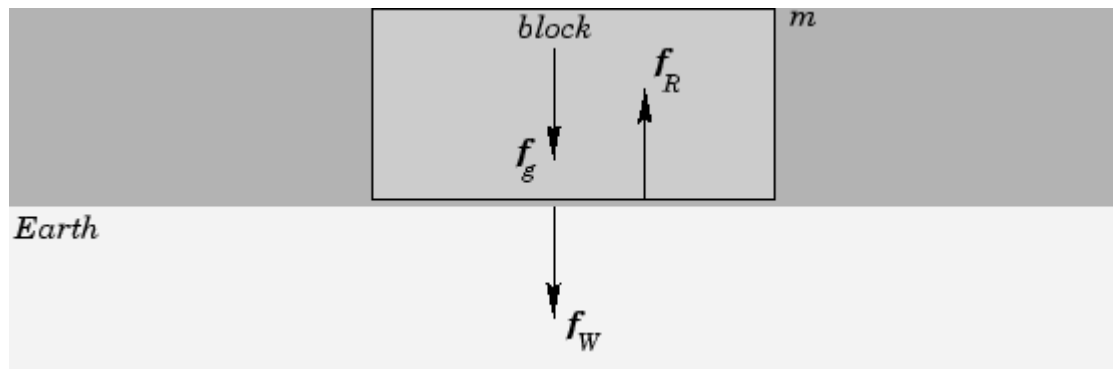


Figure 3: Weight

Imagine a block of iron resting on the surface of the Earth as seen in Figure 3. The block applies a downward force f_g due to the gravitational attraction of the Earth. The magnitude of the force is mg , where m is the mass of the block and g is the acceleration due to gravity at the surface of the Earth. The block transmits this force to the ground below it, which is supporting it, and, thereby, preventing it from accelerating downwards. In other words, the block exerts a downward force f_w , of magnitude mg , on the ground immediately beneath it. We usually refer to this force (or the magnitude of this force) as the *weight* of the block. According to Newton's third law, the ground below the block exerts an upward reaction force f_R on the block. This force is also of magnitude mg . Thus, the net force acting on the block is $f_g + f_R = 0$, which accounts for the fact that the block remains stationary. Where, you might ask, is the equal and opposite reaction to the force of gravitational attraction f_g exerted by the Earth on the block of iron? It turns out that this reaction is exerted at the centre of the Earth. In other words, the Earth attracts the block of iron, and the block of iron attracts the Earth by an equal amount. However, since the Earth is far more massive than the block, the force exerted by the iron block at the centre of the Earth has no observable consequence. So far, we have established that the weight W of a body is the magnitude of the downward force it exerts on any object, which supports it. Thus, $W = mg$ where m is the mass of the body and g is the local acceleration due to gravity. Since weight is a force, it is measured in *Newton*. A body's weight is location dependent, and is not, therefore, an intrinsic property of that body. For instance, a body weighing $10N$ on the surface of the Earth will only weigh about $3.8N$ on the surface of Mars, due to the weaker surface gravity of Mars relative to the Earth.

Lecture 5: Newton's Laws of Motion (Particle Dynamics I)

Consider a block of mass m resting on the floor of an elevator, as shown in Figure 4. Suppose that the elevator is accelerating upwards with acceleration a . How does this acceleration affect the weight of the block? Of course, the block experiences a downward force mg due to gravity. Let W be the weight of the block: by definition, this is the size of the downward force exerted by the block on the floor of the elevator. From Newton's third law, the floor of the elevator exerts an upward reaction force of magnitude N on the block.

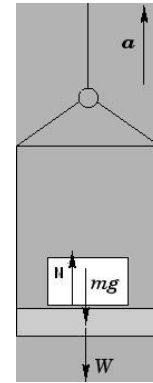


Figure 4: Weight in an elevator

Let us apply Newton's second law, Equation (5.5), to the motion of the block. The mass of the block is m , and its upward acceleration is a . Furthermore, the block is subject to two forces: a downward force mg due to gravity, and an upward reaction force N . Hence,

$$\vec{N} + \vec{W} = m\vec{a} \quad (5.6)$$

then

$$N - mg = ma \quad (\text{on the y-axis}) \quad (5.7)$$

This equation can be rearranged to give

$$N = m(g + a) \quad (5.8)$$

where the apparent weight will be greater than the actual weight. Clearly, it means that the upward acceleration of the elevator has the effect of increasing the weight W of the block: for instance, if the elevator accelerates upwards at $g = 9.81 \text{ m/s}^2$ then the weight of the block is doubled. Conversely, if the elevator accelerates downward (*i.e.*, if a becomes negative) then the weight of the block is reduced: for instance, if the elevator accelerates downward at $g/2$ then the weight of the block is halved. Incidentally, these weight changes could easily be measured by placing some scales between the block and the floor of the elevator.

Suppose that the downward acceleration of the elevator matches the acceleration due to gravity, *i.e.* $a = -g$. In this case, we will write $W = 0$. In other words, the block becomes weightless! This is the principle behind the so-called "Vomit Comet" used by NASA's Johnson Space Centre to train prospective astronauts in the effects of weightlessness. The "Vomit Comet" is actually a KC-135 (a predecessor of the *Boeing 707* which is typically used for refueling military aircraft). The plane typically ascends to $30,000 \text{ ft}$ and then accelerates downwards at g (*i.e.*, drops like a stone) for about 20 s , allowing its passengers to feel the effects of weightlessness during this period. All of the weightless scenes in the film *Apollo 11* were shot in this manner.

Finally, let us suppose that the downward acceleration of the elevator exceeds the acceleration due to gravity: *i.e.* $a < -g$. In this case, the block acquires a negative weight! What actually happens is that the block flies off the floor of the elevator and slams into the ceiling (if there is a ceiling!☺). That means: When things have settled down, the block exerts an upward force (negative weight) $|W|$ on the ceiling of the elevator.