

Name	
Surname	
Student ID	
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Question	Mark
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TOTAL	

- **CHEATING** is a **SERIOUS OFFENCE** and may lead to your **DISMISSAL** from the **UNIVERSITY!**
- The steps of solution of each problem should be shown clearly in the space given.
- Write your final result in a box and Numerical answers must be given with correct SI units.
- Take $g = 9.8 \text{ m/s}^2$, $\pi = 3,14$

QUESTION 1 (20 %)

A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00 \text{ j m/s}^2$ and an initial velocity of $\mathbf{v}_i = 5.00 \text{ i m/s}$. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00 \text{ s}$.

Solution:

$$\mathbf{a} = 3.00 \hat{\mathbf{j}} \text{ m/s}^2; \mathbf{v}_i = 5.00 \hat{\mathbf{i}} \text{ m/s}; \mathbf{r}_i = 0 \hat{\mathbf{i}} + 0 \hat{\mathbf{j}}$$

$$(a) \quad \mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2 = \boxed{5.00 t \hat{\mathbf{i}} + \frac{1}{2} 3.00 t^2 \hat{\mathbf{j}}} \text{ m}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a} t = \boxed{(5.00 \hat{\mathbf{i}} + 3.00 t \hat{\mathbf{j}})} \text{ m/s}$$

$$(b) \quad t = 2.00 \text{ s}, \mathbf{r}_f = 5.00(2.00) \hat{\mathbf{i}} + \frac{1}{2} (3.00)(2.00)^2 \hat{\mathbf{j}} = (10.0 \hat{\mathbf{i}} + 6.00 \hat{\mathbf{j}}) \text{ m}$$

$$\text{so } x_f = \boxed{10.0 \text{ m}}, y_f = \boxed{6.00 \text{ m}}$$

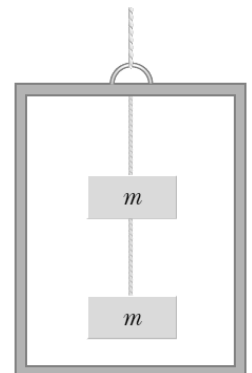
$$\mathbf{v}_f = 5.00 \hat{\mathbf{i}} + 3.00(2.00) \hat{\mathbf{j}} = (5.00 \hat{\mathbf{i}} + 6.00 \hat{\mathbf{j}}) \text{ m/s}$$

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$$

QUESTION 2 (20 %)

Two blocks, each of mass $m = 3.50 \text{ kg}$, are hung from the ceiling of an elevator cabin as in the Figure.

- If the elevator cabin moves with an upward acceleration of magnitude 1.60 m/s^2 , find the tensions in strings.
- If the strings could withstand a maximum tension of 85.0 N , what maximum acceleration would the elevator have before a string breaks?



SOLUTION:

- (a) Free-body diagrams of the two blocks are shown at the right. Note that each block experiences a downward gravitational force

$$F_g = (3.50 \text{ kg})(9.80 \text{ m/s}^2) = 34.3 \text{ N}$$

Also, each has the same upward acceleration as the elevator, in this case $a_y = +1.60 \text{ m/s}^2$.

Applying Newton's second law to the lower block:

$$\Sigma F_y = m a_y \Rightarrow T_2 - F_g = m a_y$$

$$\text{or } T_2 = F_g + m a_y = 34.3 \text{ N} + (3.50 \text{ kg})(1.60 \text{ m/s}^2) = \boxed{39.9 \text{ N}}$$

Next, applying Newton's second law to the upper block:

$$\Sigma F_y = m a_y \Rightarrow T_1 - T_2 - F_g = m a_y$$

or

$$T_1 = T_2 + F_g + m a_y = 39.9 \text{ N} + 34.3 \text{ N} + (3.50 \text{ kg})(1.60 \text{ m/s}^2) = \boxed{79.8 \text{ N}}$$

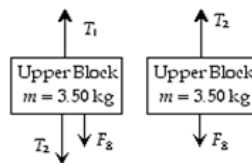
- (b) Note that the tension is greater in the upper string, and this string will break first as the acceleration of the system increases. Thus, we wish to find the value of a_y when $T_1 = 85.0 \text{ N}$.

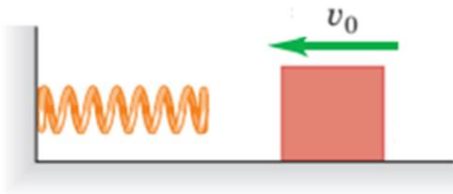
Making use of the general relationships derived in (a) above gives:

$$T_1 = T_2 + F_g + m a_y = (F_g + m a_y) + F_g + m a_y = 2F_g + 2m a_y$$

or

$$a_y = \frac{T_1 - 2F_g}{2m} = \frac{85.0 \text{ N} - 2(34.3 \text{ N})}{2(3.50 \text{ kg})} = \boxed{2.34 \text{ m/s}^2}$$



QUESTION 3 (20 %)

A 25.75-kg block is moving with a velocity of 9.55 m/s along a frictionless, horizontal surface toward a spring with a constant force 759.50 N/m which is attached to a wall. Assume that the spring has negligible mass.

- Find the maximum distance the spring will be compressed.
- If the spring is to compress by no more than 0.75 m, what should be the maximum value of initial velocity?

SOLUTION :

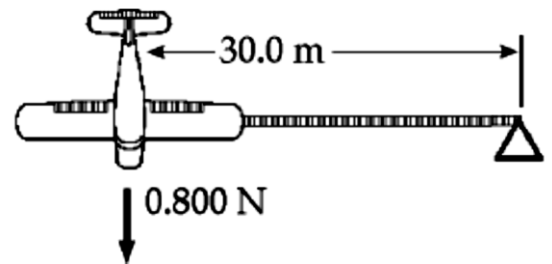
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\text{a.) } x = \sqrt{\frac{m}{k}} v = \sqrt{\frac{25.75}{759.50}} 9.55 = 1.7584 = 1.76\text{m}$$

$$\text{b.) } v = \sqrt{\frac{k}{m}} x = \sqrt{\frac{759.50}{25.75}} 0.750 = 4.0732 = 4.03\text{m/s}$$

QUESTION 4 (20 %)

A model airplane with mass 0.750 kg is connected by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.80 N perpendicular to the binding wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane when it is in level flight. (c) Find the linear acceleration of the airplane tangent to its flight path. ($I = mR^2$)

**SOLUTION**

$$m = 0.75 \text{ kg, } F = 0.80 \text{ N}$$

$$\text{(a) } \tau = rF = (30 \text{ m})(0.80 \text{ N}) = 24 \text{ N}\cdot\text{m}$$

$$\text{(b) } \alpha = \tau / I = \tau / mr^2 = (24 \text{ N}\cdot\text{m}) / [(0.75 \text{ kg})(30 \text{ m})^2] = 0.0356 \text{ rad/s}^2$$

$$\text{(c) } a_t = \alpha r = (0.0356 \text{ rad/s}^2)(30 \text{ m}) = 1.07 \text{ m/s}^2$$

$$\text{alternatively from: } a_t = F/m = 0.80 \text{ N} / 0.75 \text{ kg} = 1.07 \text{ m/s}^2$$

QUESTION 5 (20 %)

A 1.50 kg mass on a spring has a displacement (cm) as a function of time (seconds) given by the equation

$$x(t) = 7.40 \cos(4.16 t - 2.42)$$

Find (a) the period of the oscillation, (b) the force constant of the spring, (c) the maximum speed of the mass, (d) the position of the mass at $t = 1.00$ s.

Solution:

$$A = 7.40 \text{ cm, } \omega = 4.16 \text{ rad/s, and } \phi = -2.42 \text{ rad.}$$

$$\text{(a) } T = \frac{2\pi}{\omega} = \frac{2\pi}{4.16 \text{ rad/s}} = 1.51 \text{ s.}$$

$$\text{(b) } \omega = \sqrt{\frac{k}{m}} \text{ so } k = m\omega^2 = (1.50 \text{ kg})(4.16 \text{ rad/s})^2 = 26.0 \text{ N/m}$$

$$\text{(c) } v_{\max} = \omega A = (4.16 \text{ rad/s})(7.40 \text{ cm}) = 30.8 \text{ cm/s}$$

$$\begin{aligned} \text{(d) } x(t) &= (7.40 \text{ cm})\cos[(4.16 \text{ s}^{-1})t - 2.42] \\ x(t=1\text{s}) &= (7.40 \text{ cm})\cos[(4.16 \text{ s}^{-1})1 - 2.42] \\ x(t=1\text{s}) &= -1.25\text{cm} = -0.0125\text{m} \end{aligned}$$