

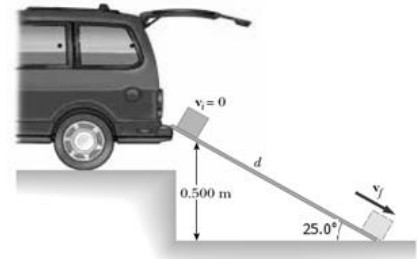
Question	Mark
1	/
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<b>TOTAL</b>	/

Name	/
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Department	/

• **CHEATING** is a **SERIOUS OFFENCE** and may lead to your **DISMISSAL** from the **UNIVERSITY!**  
 • The steps of solution of each problem should be shown clearly in the space given.  
 • Write your final result in a box and Numerical answers must be given with correct **SI units**.  
 • Take  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3,14$

### QUESTION 1 (20 %)

A 3.00-kg crate slides down a ramp. The ramp is  $d=1.18\text{m}$  in length, 0.50 m height and inclined at an angle of  $25.0^\circ$ , as shown in the figure. The crate starts from rest at the top of the ramp, experiences a constant friction force of magnitude 5.00 N as it moves downward along the ramp. Use energy methods to determine



- the initial mechanical energy (in Joules) of the crate at the top of the ramp,
- the speed (in m/s) of the crate at the bottom of the ramp,
- the final mechanical energy (in Joules) of the crate at the bottom of the ramp. ( $g=9.8 \text{ m/s}^2$ )

#### Solution:

a)

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$

$$= (3.00 \text{ kg})(9.80 \text{ m/s}^2)(0.500 \text{ m}) = 14.7 \text{ J}$$

b) Using the work done by the frictional force

$$-f_k d = (-5.00 \text{ N})(1.18 \text{ m}) = -5.90 \text{ J}$$

The change in the mechanical energy of the crate when it gets the bottom of the ramp

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$$E_f - E_i = \frac{1}{2}mv_f^2 - mgy_i = -f_k d$$

or

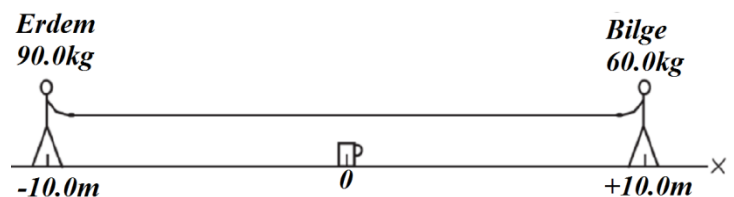
$$\frac{1}{2}mv_f^2 = 14.7 \text{ J} - 5.90 \text{ J} = 8.80 \text{ J}$$

$$v_f = 2.42 \text{ m/s}$$

c)  $E_f = \frac{1}{2}mv_f^2 = 8.80 \text{ J}$

### QUESTION 2 (20 %)

Erdem (mass 90.0 kg) and Bilge (mass 60.0 kg) are 20.0 m apart on a frozen lake. A cup of milk is between them. They pull on the ends of a light and massless rope stretched between them. When Erdem has moved 6.0 m toward the cup, how far and in what direction has Bilge moved due to cup?



#### SOLUTION:

The initial x-coordinate of the center of mass of Erdem and Bilge for their initial positions is

$$x_{\text{cm}} = \frac{(90.0 \text{ kg})(-10.0 \text{ m}) + (60.0 \text{ kg})(10.0 \text{ m})}{90.0 \text{ kg} + 60.0 \text{ kg}} = -2.0 \text{ m}$$

When Erdem moves to -4m, the center of mass does not move

$$x_{\text{cm}} = \frac{(90.0 \text{ kg})(-4.0 \text{ m}) + (60.0 \text{ kg})x_2}{90.0 \text{ kg} + 60.0 \text{ kg}} = -2.0 \text{ m}$$

$$x_2 = 1.0 \text{ m}$$

So, the final position of Bilge is 1.0 m away from the cup!

### QUESTION 3 (20 %)

A 5.00-g bullet is shot through a 1.2-kg wood block suspended on a string 2.0 m long. The center of mass of the block rises a distance of 2.0 cm. If the initial speed of the bullet was 600 m/s,

- What is the speed of the block after the collision?
- What is the speed of the bullet as it emerges from the block?

#### SOLUTION:

Let +x be to the right. Let the bullet be A and the block be B. Let  $V_B$  be the velocity of the block just after the collision.

a) Apply conservation of energy to the motion of the block after the collision:

$$K_B = U_B; \quad \frac{1}{2} m_B V_B^2 = m_B g h; \quad V_B = \sqrt{2gh} = \sqrt{2(9.8)(0.02)} = 0.63 \text{ m/s}.$$

b) Apply conservation of momentum to the collision:

$$V_{Af} = \frac{m_A V_A - m_B V_{Bf}}{m_A} = \frac{(5 \times 10^{-3} \text{ kg})(600 \frac{\text{m}}{\text{s}}) - (1.2 \text{ kg})(0.63 \text{ m/s})}{5 \times 10^{-3} \text{ kg}} = 448.8 \text{ m/s}$$

### QUESTION 4 (20 %)

Calculate the kinetic energy of a 200 kg flywheel that has the form of a hollow cylinder with inner radius 1.2 m and outer radius 1.7 m spinning at 159 rpm.

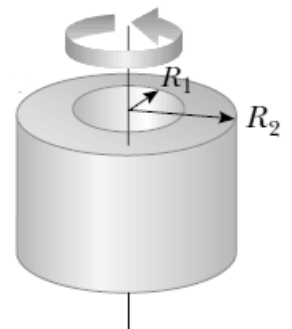
#### SOLUTION:

Hollow cylinder

$$I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$$

$E = K_E = \frac{1}{2} I \omega^2$   
 $I = \frac{1}{2} m (R_1^2 + R_2^2) = 433 \text{ kg m}^2$   
 $\omega = ?$   
 $159 \frac{\text{rev}}{\text{min}} \Rightarrow \frac{159}{60} \times 2\pi = 16.64 \text{ rad/s}$   
 $\omega^2 = 276.96$

$E = K_E = \frac{1}{2} I \omega^2$   
 $= \frac{1}{2} \times 433 \times 276.96$   
 $E_K = 59.96 \approx 60 \text{ kJ}$



### QUESTION 5 (20 %)

The angular coordinate of a rolling ball with radius 40 cm is given as  $\theta(t) = 25 + 3t + 4t^2 - 2t^3$  (rad) where t is in second. Find

- the average angular velocity and acceleration between  $t_1 = 0$  and  $t_2 = 1$  s,
- the linear acceleration of the object at  $t = 3$  s.

#### SOLUTION:

a)  $\theta(t) = 3t + 4t^2 - 2t^3 + 25 \rightarrow \theta(0) = 25 \text{ rad}$  and  $\theta(1) = 30 \text{ rad}$

$$\omega = \frac{d\theta}{dt} = 3 + 8t - 6t^2 \rightarrow \omega(0) = 3 \frac{\text{rad}}{\text{s}} \text{ and } \omega(1) = 5 \frac{\text{rad}}{\text{s}}$$

$$\bar{\omega} = \frac{\theta(1) - \theta(0)}{1 - 0} = \frac{30 - 25}{1 - 0} = 5 \text{ rad/s}$$

$$\bar{\alpha} = \frac{\omega(1) - \omega(0)}{1 - 0} = \frac{5 - 3}{1 - 0} = 2 \frac{\text{rad}}{\text{s}^2}$$

b)

$$\alpha = \frac{d^2\theta}{dt^2} = 8 - 12t \rightarrow \alpha(3) = -28 \frac{\text{rad}}{\text{s}^2} \text{ and } a = \alpha R = -28 \frac{\text{rad}}{\text{s}^2} * 0.4 \text{ m} = -11.2 \frac{\text{m}}{\text{s}^2}$$