

Name-Surname	
Number	
Signature	

Date: 23/11/2021 Time: 17:00 Duration: 90 min.

- Cheating is a serious offence and may lead to your dismissal from the university.
- Write clearly your solutions steps to the answer book.
- Solutions in the question paper will not be taken into account.
- Constants:  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3.14$

### QUESTION 1 (20 %)

Consider two vectors are defined as  $\vec{A} = -3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$  and  $\vec{B} = 6\mathbf{i} + m\mathbf{j} - 10\mathbf{k}$ . Determine,

- the constant  $m$  that satisfies the cross product of two vectors is zero, and
- the angle between two vectors.

### SOLUTION

a)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 5 \\ 6 & m & -10 \end{vmatrix} = (20 - 5m)\mathbf{i} - (30 - 30)\mathbf{j} + (-3m + 12)\mathbf{k}$$
$$(20 - 5m) = 0 \text{ and } (-3m + 12) = 0 \rightarrow m = 4$$

b)  $\vec{A} \cdot \vec{B} = AB \cos(\theta) \rightarrow \theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{AB} \right) = \cos^{-1} \left( \frac{(-3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) \cdot (6\mathbf{i} + 4\mathbf{j} - 10\mathbf{k})}{(\sqrt{9+4+25} \sqrt{36+16+100})} \right) = \cos^{-1}(-1) = 180^\circ$

### QUESTION 2 (20 %)

A stone is thrown vertically upward from a point near the top edge of a tall building. It just misses the edge on the way down and passes a point 50 m below its starting point 5 s after it leaves the thrower's hand. Air resistance may be ignored. a) What is the initial speed of the stone? b) How high does it rise above its starting point? c) What is the magnitude of its velocity at the highest point?

### SOLUTION

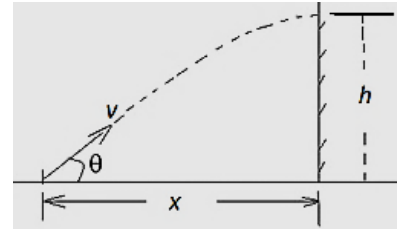
a)  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 \rightarrow v_{0y} = \frac{y - y_0}{t} - \frac{1}{2}a_y t = \frac{-50.0 \text{ m}}{5.00 \text{ s}} - \frac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s}) = +14.5 \text{ m/s}.$

b)  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \rightarrow y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (14.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 10.7 \text{ m}$

c) At the maximum height  $v_y = 0.$

**QUESTION 3 (20 %)**

A projectile is launched from the ground at an angle  $\theta=45^\circ$  to the horizontal and initial speed  $v=21$  m/s. At what height does the projectile hit a vertical wall that is  $x=9.0$  m away? (Neglect air resistance,  $\sin 45 = \cos 45 = 1/\sqrt{2}$ )

**SOLUTION**

$$y = (\tan \theta) x - \left( \frac{g}{2v^2 \cos^2 \theta} \right) x^2 = \underline{7.2 \text{ m}}$$

Solution 2 (from 1D motion)

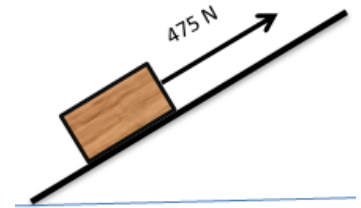
$$x = v(\cos \theta) t = 21 \cos 45 t \Rightarrow t = \frac{9}{21 \cos 45}$$

$$y = v(\sin \theta) t - \frac{1}{2} g t^2$$

$$= 21(\sin 45) t - \frac{1}{2} \cdot 9.8 \times t^2 = \underline{7.2 \text{ m}}$$

**QUESTION 4 (20 %)**

A 55.0 kg crate pulled 11.0 m up a  $44.0^\circ$  ramp by a weightless string with a constant tension 475 N. The coefficient of kinetic friction is 0.33 between the crate and the surface. Calculate the work done by each force acting on the crate.

**SOLUTION**

$$F_g = m_2 g$$

$$F_g = 539.0 \text{ -j N}$$

$$F_N = F_g \cos 44^\circ = 387.72$$

$$F_N = 387.7 \text{ j N}$$

$$F_f = F_N \mu = 127.94$$

$$F_f = 127.9 \text{ -j N}$$

$$W_F = Fd = Fd \cos 0 = 5225$$

$$W_F = 5.2 \text{ kJ}$$

$$W_{F_g} = F_g d \cos 134^\circ = F_g d \cos 134^\circ = -4118.6 \text{ J}$$

$$W_{F_g} = -4.1 \text{ kJ}$$

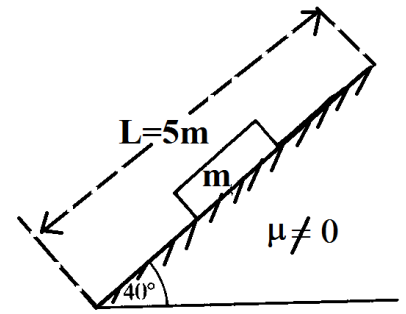
$$W_{F_f} = F_f \times d = F_f d \cos 180^\circ = -1406.9 \text{ J}$$

$$W_{F_f} = -1.4 \text{ kJ}$$

### QUESTION 5 (20 %)

A rectangular box of  $m=12$  kg is released from the top of an inclined plane that is  $L=5.0$  m long and makes an angle of  $40^\circ$  to the horizontal. If a constant friction force of  $60.0$  N opposes the motion of the box, then

- Draw the Free-Body Diagram of the system on your answer sheet.
- What is the acceleration of the mass?
- What is the magnitude of the friction coefficient between the box and incline?
- What would be the speed of the box if it reached the bottom of incline?



### SOLUTION

a)

b)

$$y: N = mg \cos \theta$$
$$x: mg \sin \theta - F_f = ma$$
$$118 \times 0.642 - 60 = 12a$$
$$a = \underline{\underline{1.31 \text{ m/s}^2}}$$

c)

$$F_f = \mu N \Rightarrow 60 = \mu mg \cos \theta$$
$$\mu = \frac{60}{118 \times 0.766} \Rightarrow \underline{\underline{\mu = 0.66}}$$

d)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$5 = 0 + 0 + \frac{1}{2} (1.31) t^2$$
$$t = \underline{\underline{2.76 \text{ s}}}$$

$$v = v_0 + a t$$
$$v = 0 + 1.31 \times 2.76 \Rightarrow \underline{\underline{v = 3.62 \text{ m/s}}}$$