



Date: 09/08/2019 Time: 13:30 Duration: 90 min.

Ques.	Mark
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Total	

EDUCATION : 1st Ed. 2nd Ed.
DEPARTMENT : CE MME IE ME TE

Name	Surname	Student No	Signature

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants: $g = 9.8 \text{ m/s}^2$, $\pi = 3.141593$
- $1 \text{ mm} = 10^{-3} \text{ m}$, $1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ nm} = 10^{-9} \text{ m}$, $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ h} = 3600 \text{ s}$, $1 \text{ min} = 60 \text{ s}$, $1 \text{ rev} = 2\pi \text{ rad}$.

QUESTION 1 (20 %)

A billiard ball moving at 5.0 m/s collides elastically with another ball of the same mass and initially at rest. Two balls have equal speeds after the collision. Find the speed of each ball after collision and determine the direction (angle) of the velocity vectors after collision.

- K-E is conserved in collision since the collision is elastic.

$$\frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} m v_f^2$$

Solving for v_f , we obtain

$$v_f = \frac{v_i}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.5 \text{ m/s}$$

- Conservation of momentum:

in x-axis: $m v_i = m v_f \cos \theta_1 + m v_f \cos \theta_2$ (1)

in y-axis: $0 = m v_f \sin \theta_1 - m v_f \sin \theta_2$ (2)

From eqn(2), we see $\theta_1 = \theta_2 = \theta$

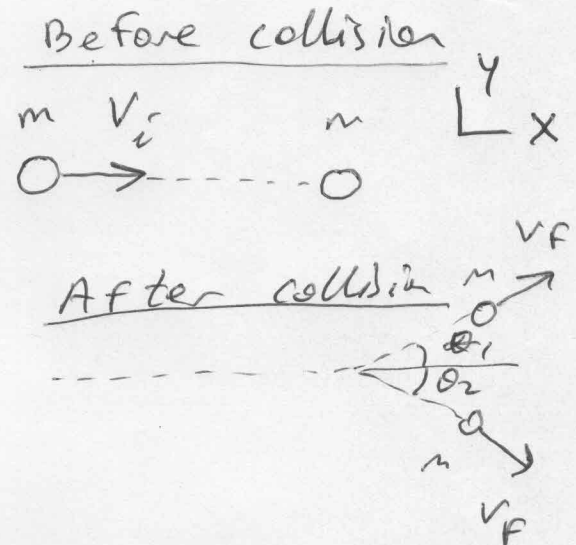
From eqn(1) $\cos \theta = \frac{v_i}{2 v_f} = \frac{5}{2(3.5)} = \frac{1}{\sqrt{2}} \rightarrow \theta = 45^\circ$

$v_{1f} = 3.5 \text{ m/s}$

$v_{2f} = 3.5 \text{ m/s}$

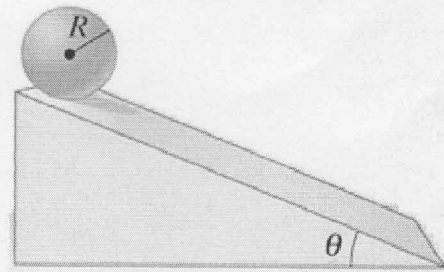
$\theta_1 = 45^\circ$

$\theta_2 = 45^\circ$



QUESTION 2 (20 %)

A hollow, spherical shell with mass 2.0 kg rolls without slipping down a $\theta = 38^\circ$ slope as shown in figure right [Rotational inertia of the ball about its center of mass is given by: $I = 2mR^2 / 3$].



(a) Calculate the linear acceleration of the spherical shell as it rolls down.

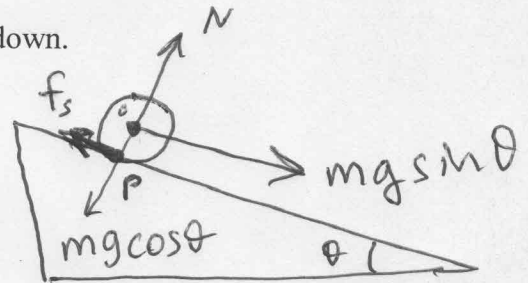
$$I_p = I_{cm} + mR^2 = \frac{2}{3}mR^2 + mR^2 = \frac{5}{3}mR^2$$

Torque about point P:

$$\tau_p = (mg \sin \theta) R = I_p \alpha = \frac{5}{3} mR^2 \left(\frac{a}{R}\right)$$

Solving for a:

$$a = \frac{3}{5} g \sin \theta = \frac{3}{5} (9.8) \sin 38 = 3.6 \text{ m/s}^2$$



$a = 3.6 \text{ m/s}^2$

(b) Calculate the frictional force between the inclined plane and the shell.

Torque about cm:

$$\tau_o = f_s R = I_{cm} \alpha = I_{cm} \frac{a}{R} = \frac{2}{3} mR^2 \left(\frac{a}{R}\right)$$

Solving f_s :

$$f_s = \frac{2}{3} ma = \frac{2}{3} (2) (3.6) = 4.8 \text{ N}$$

OR

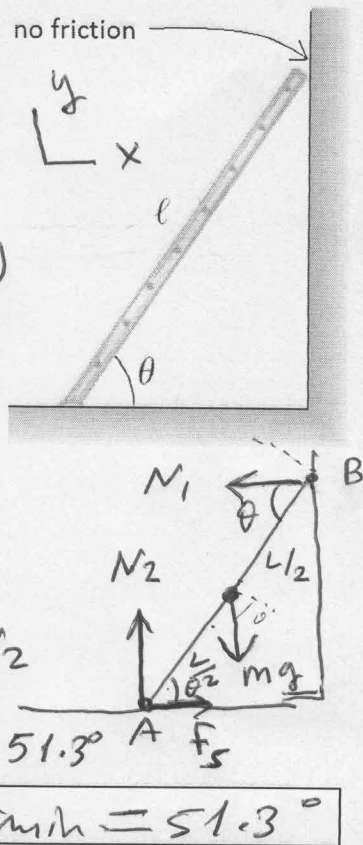
$$F_{net} = ma$$

$$mg \sin \theta - f_s = ma \Rightarrow f_s = mg \sin \theta - ma = 4.8 \text{ N}$$

$f_s = 4.8 \text{ N}$

QUESTION 3 (20 %)

A uniform ladder of length L rests against a smooth, vertical wall as shown in Figure. The mass of the ladder is m , and the coefficient of static friction between the ladder and the ground is $\mu = 0.4$. Find the minimum angle (θ_{\min}) at which the ladder does not slip.



$$\bullet \sum F_x = 0 \rightarrow -N_1 + F_s = 0 \rightarrow F_s = N_1 \quad (1)$$

$$\bullet \sum F_y = 0 \rightarrow -mg + N_2 = 0 \rightarrow N_2 = mg \quad (2)$$

$$\bullet \sum \tau_A = 0 \rightarrow N_1 \sin \theta L - mg \cos \theta \frac{L}{2} = 0$$

$$N_1 \sin \theta = \frac{mg \cos \theta}{2}$$

$$\text{or } \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{mg}{2N_1} = \frac{mg}{2F_s} = \frac{mg}{2\mu N_2}$$

$$\therefore \tan \theta = \frac{mg}{2\mu mg} = \frac{1}{2\mu} = \frac{1}{0.8} \Rightarrow \theta = 51.3^\circ$$

$\theta_{\min} = 51.3^\circ$

QUESTION 4 (20 %)

A mass is attached to spring and executes a simple harmonic motion. The spring constant is $k = 50 \text{ N/m}$. The position of the mass varies according to the expression:

$$x(t) = (0.5 \text{ m}) \cos(2t)$$

where x is in meters and t is in seconds.

(a) Find the speed of the mass at $t = 2 \text{ s}$.

$$v(t) = \frac{dx}{dt} = \frac{d}{dt} [0.5 \cos 2t] = -\sin(2t)$$

$$v(2) = -\sin(4) = -0.757 \text{ m/s}$$

$v = -0.757 \frac{\text{m}}{\text{s}}$

(b) Find the acceleration of the mass at $t = 2 \text{ s}$.

$$a(t) = \frac{dv}{dt} = \frac{d}{dt} [-\sin(2t)] = -2 \cos(2t)$$

$$a(2) = -2 \cos(4) = +1.307 \text{ m/s}^2$$

$a = 1.307 \text{ m/s}^2$

(c) Find the kinetic energy of the mass at $t = 2 \text{ s}$.

$$K = \frac{1}{2} m v^2 = \frac{1}{2} (12.5) (-0.757)^2 = 3.582 \text{ J}$$

$K = 3.582 \text{ J}$

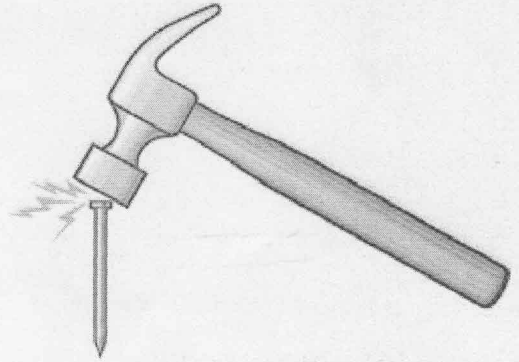
(d) Find the total potential energy of the mass at $t = 2 \text{ s}$.

$$U = \frac{1}{2} k x^2 = \frac{1}{2} (50) \left[\underbrace{0.5 \cos(4)}_{x(2)} \right]^2 = 2.670 \text{ J}$$

$U = 2.670 \text{ J}$

QUESTION 5 (20 %)

A 12-kg hammer strikes a nail at a velocity of 7.5 m/s and comes to rest in a time interval of 8.0 ms. (1 ms = 10^{-3} s).



(a) What is the impulse given to the nail?

Impulse \equiv change in momentum

$$\begin{aligned} J &= \Delta p = p_f - p_i \\ &= m v_f - m v_i \\ &= 0 - (12)(7.5) \\ &= -90 \text{ kg}\cdot\text{m/s} \end{aligned}$$

$$J = -90 \text{ kg}\cdot\frac{\text{m}}{\text{s}}$$

(b) What is the average force acting on the nail?

$$J = \int_0^t F dt = F_{\text{avr}} \Delta t = \Delta p$$

or

$$F_{\text{avr}} = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} = \frac{-90 \text{ kg}\cdot\text{m/s}}{8 \times 10^{-3} \text{ s}}$$

$$\begin{aligned} F_{\text{avr}} &= 11250 \text{ N} \\ &= 11.25 \text{ kN} ! \end{aligned}$$

$$F_{\text{avr}} = 11250 \text{ N}$$