



Date: 07/01/2020 Time: 10:30 Duration: 90 min.

DEPARTMENT : CE MME IE ME TE

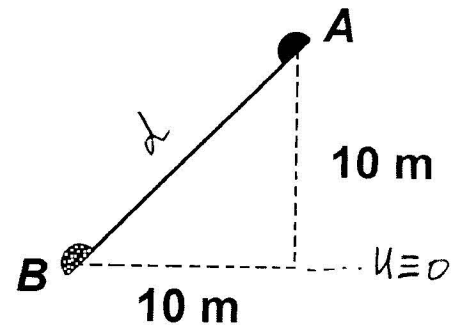
Name	Surname	Student No	Signature

Ques.	Mark
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- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants: $g = 9.8 \text{ m/s}^2$, $\pi = 3.141593$
- Conversions: $1 \text{ g} = 10^{-3} \text{ kg}$, $1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ h} = 3600 \text{ s}$, $1 \text{ min} = 60 \text{ s}$, $1 \text{ rev} = 2\pi \text{ rad}$.

QUESTION - 1 (20 %)

A 10 kg mass slides from position *A* to *B* on an incline as shown in the figure. If the body is initially at rest, what is the speed at point *B* of the body, if;



a) the incline is frictionless?

Conservation of *M-E*.

$$E_i = E_f$$

$$\frac{1}{2} m v^2 = mgh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2(9.8)(10)} = 14.0 \text{ m/s}$$

$V = 14 \text{ m/s}$

b) the incline exerts a frictional force of 34 N on the body?

Work done by friction = change in *M-E*

$$W_f = \Delta E = E_f - E_i$$

$$-fd = \frac{1}{2} m v^2 - mgh$$

$$v = \sqrt{\frac{2}{m} (mgh - fd)}$$

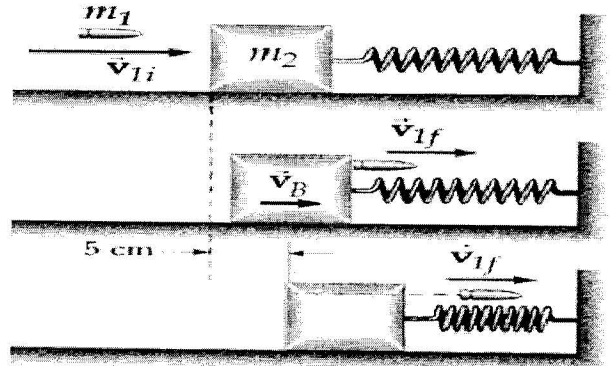
$$= \left[\frac{2}{10} (10 \times 9.8 \times 10 - 34 \times 10 \sqrt{2}) \right]^{1/2}$$

$$= 10 \text{ m/s}$$

$V = 10 \text{ m/s}$

QUESTION - 2 (20 %)

A bullet of mass $m_1=5.00$ g fired horizontally at a speed of $v_{1i}=400$ m/s and it hits a $m_2=1.00$ kg block initially at rest on a frictionless, horizontal surface. The block is also connected to a spring with force constant $k=900$ N/m. The bullet hits and passes through the block and the block moves $d=5.00$ cm to the right, after the collision, before being brought to rest by the spring as shown in symbolic Figures. Assume that the collision occurs on a line through the center of masses.



a) Find the speed of the block, V_B , after the collision.

Conservation of Energy:

$$(K+U)_{\text{before}} = (K+U)_{\text{after}}$$

$$\frac{1}{2} m v_B^2 = \frac{1}{2} k x^2$$

or

$$v_B = \sqrt{\frac{k}{m_2}} x = \sqrt{\frac{900}{1}} \cdot 0.05 = 1.5 \text{ m/s}$$

$$V_B = 1.5 \text{ m/s}$$

b) Find the speed, v_{1f} , at which the bullet emerges from the block after the collision.

Conservation of momentum:

$$m_1 v_{1i} + \underbrace{m_2 v_{2i}}_0 = m_1 v_{1f} + m_2 v_{2f} \quad \leftarrow v_{2f} = v_B$$

Solving for v_{1f} :

$$v_{1f} = \frac{m_1 v_{1i} - m_2 v_B}{m_1}$$

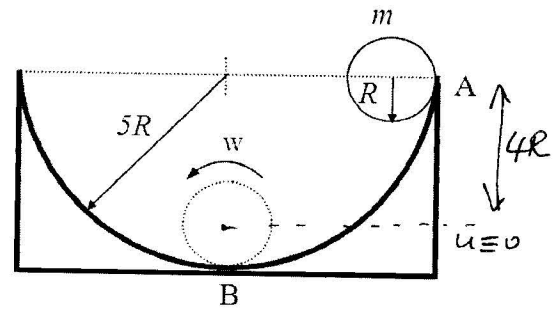
$$= \frac{(5 \times 10^{-3})(400) - (1)(1.5)}{5 \times 10^{-3}}$$

$$= 100 \text{ m/s}$$

$$v_{1f} = 100 \text{ m/s}$$

QUESTION - 3 (20 %)

A wheel of tractor have a mass of $m = 30 \text{ kg}$ and radius $R = 70 \text{ cm}$. It rolls without slipping in a cylindrical trough of radius $5R$ as shown in Figure. The wheel is released from point A and passes through at the bottom of the trough (point B) with speed of 7 m/s .



a) Find the moment of inertia of wheel

Conservation of $M \cdot \dot{E}$.

$$E_i = E_f$$

$$4mgR = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$4mgR = \frac{1}{2} m v^2 + \frac{1}{2} I \frac{v^2}{R^2}$$

Solving for I :

$$I = \frac{R^2}{v^2} (8mgR - mv^2)$$

$$= \frac{0.7^2}{7^2} (8(30)(9.8)(0.7) - (30)(7)^2) = 1.76 \text{ kg} \cdot \text{m}^2$$

$I = 1.76 \text{ kg} \cdot \text{m}^2$

b) Find the angular momentum of the sphere about its center of mass at point B.

Angular momentum about CM

$$L_{cm} = I_{cm} \omega = I_{cm} v/R$$

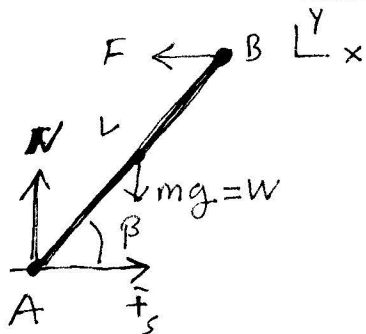
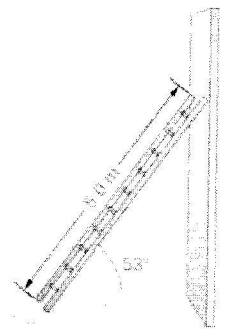
$$= (1.76)(7/0.7)$$

$$= 17.6 \text{ kg} \cdot \text{m}^2/\text{s}$$

$L = 17.6 \text{ kg} \cdot \text{m}^2/\text{s}$

QUESTION - 4 (20 %)

A uniform ladder is $L = 5.0 \text{ m}$ long and weighs 400 N . The ladder rests against a slippery vertical wall, as shown in Figure. The inclination angle between the ladder and the rough floor is $\beta = 53^\circ$. Find the coefficient of static friction μ_s at the interface of the ladder with the floor that prevents the ladder from slipping.



Torque about point A

$$-\frac{L}{2} W \cos \beta + LF \sin \beta$$

$$F = \frac{W}{2} \cos \beta = \frac{400}{2} \cos 53^\circ = 150.7 \text{ N}$$

$$\mu_s = \frac{F_s}{N} = \frac{150.7}{400.0} = 0.377$$

$\mu_s = 0.377$

$$N = W$$

$$F = F_s$$

QUESTION - 5 (20 %)

An object attached to a spring of force constant 2 N/m is oscillating about $x=0$ on the x-axis. Displacement of the object as a function of time is given by $x(t) = (10 \text{ cm}) \cos (0.2\pi t + 1.3\pi)$.

a) What is the displacement of the object from the equilibrium position at $t = 1 \text{ s}$?

$$x(t) = 10 \cos (0.2\pi t + 1.3\pi)$$

$$x(1) = 10 \cos (0.2\pi + 1.3\pi)$$

$$= 10 \cos (1.5\pi)$$

$$= 0$$

$$x(1) = 0$$

b) What is the speed of the object at $t = 1 \text{ s}$?

Velocity is the time derivative of x .

$$v = \dot{x} = \frac{d}{dt} (10 \cos (0.2\pi t + 1.3\pi))$$

$$= -0.2 \times 10\pi \sin (0.2\pi t + 1.3\pi)$$

$$v(1) = -2\pi \sin (1.5\pi)$$

$$= 6.28 \text{ cm/s}$$

$$= 0.0628 \text{ m/s}$$

$$v(1) = 0.0628 \frac{\text{m}}{\text{s}}$$

c) What is the total energy of the object at $t = 1 \text{ s}$?

$$\text{Total energy is } E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$\therefore E = \frac{1}{2} k A^2 = \frac{1}{2} (2) (0.1)^2 = 0.01 \text{ J}$$

$$= 0.01 \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

$$E = 0.01 \text{ J}$$