



Date: 01/08/2019 Time: 13:30 Duration: 90 min.

Ques.	Mark
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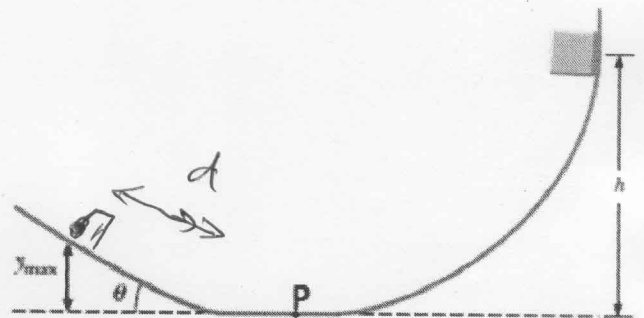
DEPARTMENT : CE  MME  IE  ME  TE

Name	Surname	Student No	Signature

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants:  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3.141593$
- Conversions:  $1 \text{ g} = 10^{-3} \text{ kg}$ ,  $1 \text{ cm} = 10^{-2} \text{ m}$ ,  $1 \text{ km} = 10^3 \text{ m}$ ,  $1 \text{ h} = 3600 \text{ s}$ ,  $1 \text{ min} = 60 \text{ s}$ ,  $1 \text{ rev} = 2\pi \text{ rad}$ .

**QUESTION 1 (20 %)**

A block of mass  $2.0 \text{ kg}$  slides down a curved frictionless track from a height  $h = 1.2 \text{ m}$  and then up an inclined plane with an angle  $\theta = 37^\circ$  as in figure. The coefficient of kinetic friction between the block and the incline is  $\mu_k = 0.4$ .



(a) What is the kinetic energy of the block at point P?

$$K_p = \frac{1}{2} m v_p^2 = mgh$$

$$= (2) (9.8) (1.2)$$

$$= 23.52 \text{ J}$$

$K_p = 23.52 \text{ J}$

(b) What is the maximum height  $y_{max}$  that is reached by the block?

$$\frac{1}{2} m v_p^2 = mg y_{max} + \mu_k (mg \cos \theta) d$$

$$K_p = mg y_{max} + \mu_k \left( mg \frac{y_{max}}{\tan \theta} \right) = \left[ 1 + \frac{\mu_k}{\tan \theta} \right] mg y_{max}$$

Solving for  $y_{max}$ :

$$y_{max} = \frac{K_p}{\left[ 1 + \mu_k / \tan \theta \right] mg} = 0.78 \text{ m}$$

$y_{max} = 0.78 \text{ m}$

### QUESTION 2 (20 %)

An object is rotating with an initial speed of 48 m/s on a circular track of diameter 2.4 km. The object slows down with a constant angular acceleration of  $-2 \times 10^{-4} \text{ rad/s}^2$ . Find

- a) the time required to take for the object to come to a stop, and

$$\omega_0 = \frac{v}{r} = \frac{48 \text{ m/s}}{2.4 \times 10^3 / 2} = 4 \times 10^{-2} \text{ rad/s}$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 4 \times 10^{-2}}{-2 \times 10^{-4}} = 200 \text{ s}$$

$$t = 200 \text{ s}$$

- b) the number of revolutions around the track the object makes before coming to a stop.

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

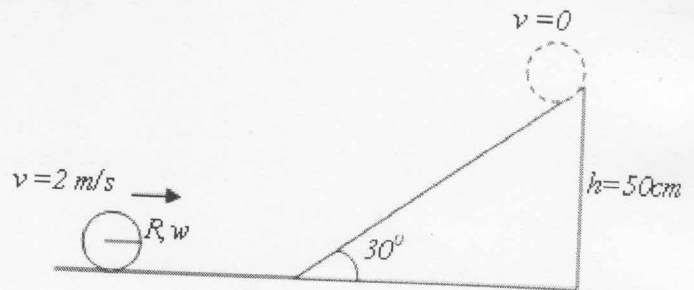
$$0 = (4 \times 10^{-2})^2 + (2)(-2 \times 10^{-4})\theta \Rightarrow \theta = 4 \text{ rad.}$$

$$\theta = 4 \text{ rad} \frac{1 \text{ rev}}{2\pi \text{ rad}} = 0.64 \text{ rev.}$$

$$\theta = 0.64 \text{ rev.}$$

### QUESTION 3 (20 %)

A bowling ball of radius  $R = 20 \text{ cm}$  and mass  $m = 10 \text{ kg}$  is rolling without slipping up a hill to a height  $50 \text{ cm}$  before momentarily coming to rest. If the initial speed of ball (at  $t=0 \text{ s}$ ) is  $2 \text{ m/s}$ , find



- (a) the moment of inertia of the ball ( $I$ ) about its center of mass, and

COE:  $E_i = E_f$

$$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = mgh$$

$$\frac{1}{2} m v^2 + \frac{1}{2} I \frac{v^2}{R^2} = mgh$$

$$\frac{1}{2} (10) (2)^2 + \frac{1}{2} I \left(\frac{2}{0.2}\right)^2 = (10)(9.8)(0.5)$$

Solving for  $I$ :

$$I = 0.58 \text{ kg}\cdot\text{m}^2$$

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- (b) the angular momentum ( $L$ ) of the ball at  $t = 0 \text{ s}$ .

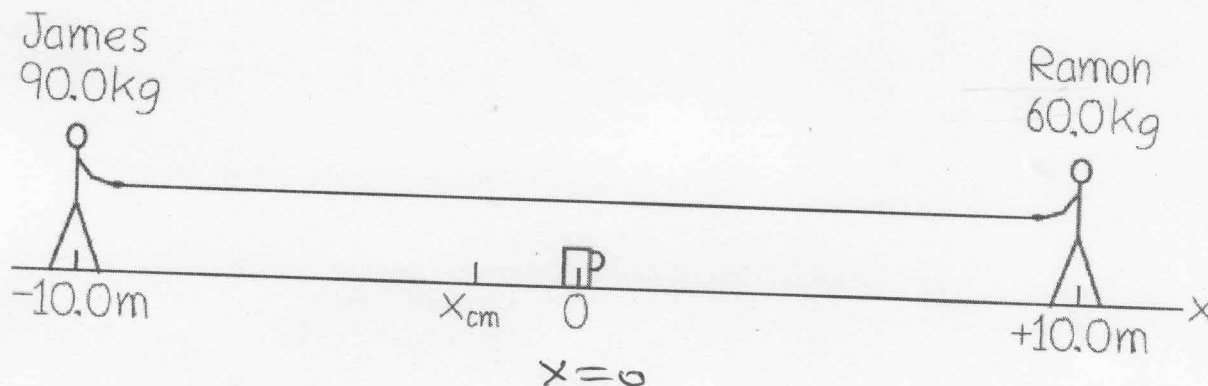
$$\text{at } t=0 \quad \omega = \frac{v}{r} = \frac{2 \text{ m/s}}{0.2 \text{ m}} = 10 \text{ rad/s}$$

$$L = I\omega = (0.58)(10) = 5.8 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$L = 5.8 \text{ kg}\cdot\text{m}^2/\text{s}$$

QUESTION 4 (20 %)

James (mass  $m_1 = 90.0$  kg) and Ramon (mass  $m_2 = 60.0$  kg) are 20.0 m apart on a icy surface. Midway between them is a cup of their tea. They pull on the ends of a light rope stretched between them.



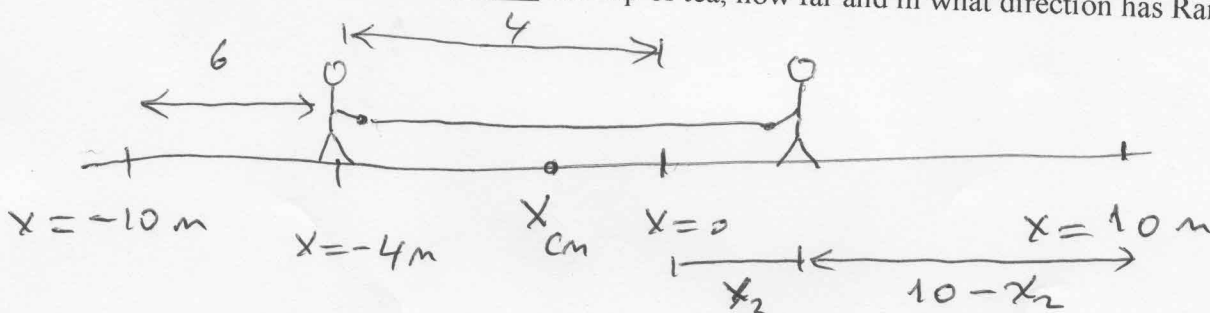
(a) Where is the center of mass of them ( $x_{cm} = ?$ )?

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(90)(-10) + (60)(10)}{90 + 60}$$

$$= -2 \text{ m}$$

$x_{cm} = -2 \text{ m}$
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(b) When James has moved 6.0 m toward the cup of tea, how far and in what direction has Ramon moved?



since there is no external force on the system, center of mass does not change. Therefore:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$-2 = \frac{(90)(-4) + (60)(x_2)}{90 + 60}$$

$9 \text{ m}$
toward James.

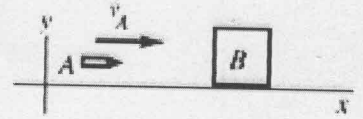
or toward cm,

Solving for  $x_2 = 1 \text{ m}$

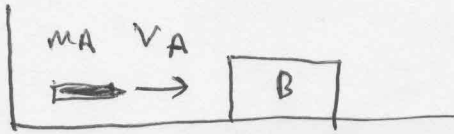
Hence Ramon moved toward James 9 m

QUESTION 5 (20 %)

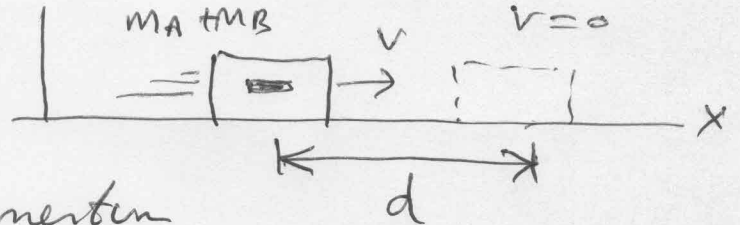
A 3.5 g bullet is fired horizontally into a 1.5 kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between the block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.150 m along the surface before stopping. What is the initial speed of the bullet?



Before collision:



After collision:



- Conservation of momentum

$$m_A v_A = (m_A + m_B) v \quad (1)$$

- Conservation of Energy:

$$K_i + U_i + W_f = K_f + U_f \quad (2)$$

$W_f \equiv$  work done by friction

$$\begin{aligned} W_f &= f_k d \cos 180^\circ \\ &= -\mu N d \\ &= -\mu (m_A + m_B) g d \end{aligned}$$

$$\frac{1}{2} (m_A + m_B) v^2 + 0 - \mu (m_A + m_B) g d = 0 + 0$$

Solving for v:

$$v = \sqrt{2 \mu g d} = \sqrt{(2)(0.2)(9.8)(0.23)} = 0.95 \text{ m/s}$$

using eqn (1)

$$v_A = \left( \frac{m_A + m_B}{m_A} \right) v = \left( \frac{5 \times 10^{-3} + 1.2}{5 \times 10^{-3}} \right) 0.95 = 229 \frac{\text{m}}{\text{s}}$$

$$v_A = 229 \text{ m/s}$$