



Date: 12/12/2019 Time: 10:20 Duration: 90 min.

DEPARTMENT : CE MME IE ME TE

Name	Surname	Student No	Signature
	SOLUTIONS		

Ques.	Mark
1	
2	
3	
4	
5	
Total	

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants: $g = 9.8 \text{ m/s}^2$, $\pi = 3.141593$
- Conversions: $1 \text{ g} = 10^{-3} \text{ kg}$, $1 \text{ cm} = 10^{-2} \text{ m}$, $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ h} = 3600 \text{ s}$, $1 \text{ min} = 60 \text{ s}$, $1 \text{ rev} = 2\pi \text{ rad}$.

QUESTION 1 (20 %)

A system is composed of three point-like particles. The masses and the corresponding positions of these particles are given as follows:

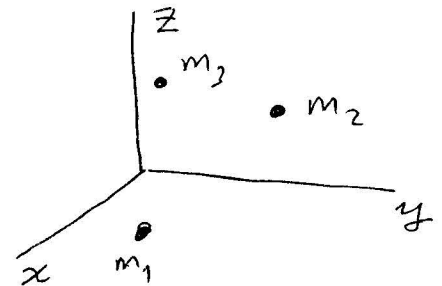
$$m_1 = 3 \text{ kg at } (0 \text{ cm}, 30 \text{ cm}, 20 \text{ cm});$$

$$m_2 = 1 \text{ kg at } (20 \text{ cm}, 40 \text{ cm}, 0 \text{ cm});$$

$$m_3 = 2 \text{ kg at } (35 \text{ cm}, 0 \text{ cm}, 55 \text{ cm}).$$

Find the center of mass coordinate of the system (as a position vector) of the particles.

$$\begin{aligned} x_{cm} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ &= \frac{(3)(0) + (1)(20) + (2)(35)}{3 + 1 + 2} \\ &= 15 \text{ cm} = 0.15 \text{ m}. \end{aligned}$$



$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(3)(30) + (1)(40) + 2(0)}{6} = 22 \text{ cm} = 0.22 \text{ m}$$

$$z_{cm} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3}{m_1 + m_2 + m_3} = \frac{(3)(20) + (1)(0) + 2(55)}{6} = 28 \text{ cm} = 0.28 \text{ m}$$

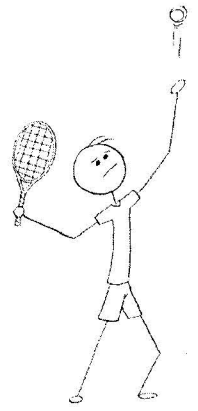
$$\begin{aligned} \vec{r}_{cm} &= x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k} \\ &= 15 \hat{i} + 22 \hat{j} + 28 \hat{k} \text{ (cm)} \end{aligned}$$

$$\text{or } \vec{r}_{cm} = 0.15 \hat{i} + 0.22 \hat{j} + 0.28 \hat{k} \text{ (m)}$$

$$\boxed{\vec{r}_{cm} = 15 \hat{i} + 22 \hat{j} + 28 \hat{k} \text{ (cm)}}$$

QUESTION 2 (20 %)

For a top player, a tennis ball may leave the racket on the serve with a speed of 55 m/s . Assume that the ball has a mass of 0.06 kg and is in contact with the racket for about $4 \times 10^{-3} \text{ s}$.



(a) What is the impulse given to the ball by the racket?

$$\begin{aligned} \text{Impulse: } J &= \Delta p = p_f - p_i \\ &= mv_f - mv_i \\ &= (0.06)(55) - 0 \\ &= 3.3 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$J = 3.3 \text{ kg} \cdot \text{m/s}^2$$

(b) Calculate the average force on the ball.

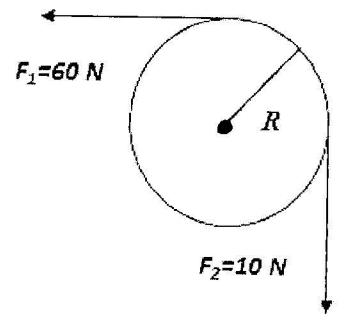
$$\begin{aligned} J &= \Delta p = \int_0^t F(t) dt = F_{\text{AVR}} \Delta t \\ F_{\text{AVR}} &= \frac{\Delta p}{\Delta t} = \frac{3.3 \text{ kg} \cdot \text{m/s}^2}{4 \times 10^{-3} \text{ s}} = 825 \text{ N} \end{aligned}$$

This force can be used to hold a man of mass 80 kg

$$F_{\text{AVR}} = 825 \text{ N}$$

QUESTION 3 (20 %)

The wheel rotates without friction about a stationary horizontal axis that passes through the center of the wheel. The wheel is a uniform disk with radius $R = 20 \text{ cm}$ and mass of $M = 10 \text{ kg}$. The system is released from rest. Two forces are applied to the wheel as in Figure with magnitude $F_1 = 60 \text{ N}$ and $F_2 = 10 \text{ N}$. (Rotational inertia of a pulley about its center of mass is $I = \frac{1}{2}MR^2$)



(a) What is the net torque acted on the wheel?

$$\tau = (F_1 - F_2) R = (60 - 10)(0.2) = 10 \text{ N} \cdot \text{m}$$

$$\tau = 10 \text{ N} \cdot \text{m}$$

(b) What are the linear and angular accelerations of the wheel?

$$\tau = I \alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{\tau}{MR^2/2} = \frac{10}{10(0.2)^2/2} = 50 \frac{\text{rad}}{\text{s}^2}$$

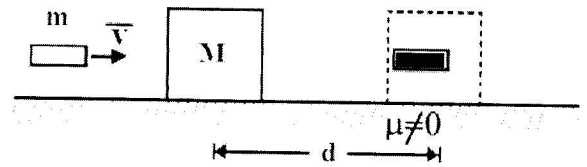
linear acceleration:

$$a = \alpha R = (50)(0.2) = 10 \frac{\text{m}}{\text{s}^2}$$

$$\begin{aligned} \alpha &= 50 \text{ rad/s}^2 \\ a &= 10 \text{ m/s}^2 \end{aligned}$$

QUESTION 4 (20 %)

A 4.0 g bullet is fired horizontally with a speed of 300 m/s into a 0.8 kg wooden block initially at rest on a table, and the bullet has embedded into the block in the collision. The coefficient of friction between the block and the table is 0.3.



(a) How far will the block slide on the table after the collision?

- Conservation of momentum:

$$m v_B = (m + M) V$$

$$(0.004)(300) = (0.004 + 0.8) V$$

Solving for V yields $V = 1.493 \text{ m/s}$ ($v \approx 1.5 \frac{\text{m}}{\text{s}}$)

- After collision, the frictional force:

$$f = \mu (m + M) g$$

- Work-energy theorem:

$$-f_s = \Delta K = K_f - K_i$$

$$-\mu (m + M) g s = 0 - \frac{1}{2} (m + M) V^2$$

$$s = \frac{V^2}{2\mu g} = \frac{(1.493)^2}{(2)(0.3)(9.8)} = 0.379 \text{ m} \quad (s \approx 0.4 \text{ m})$$

$$s = 0.379 \text{ m}$$

(b) What is the percentage change in the kinetic energy of the bullet just after the collision?

Fractional K.E. lost:

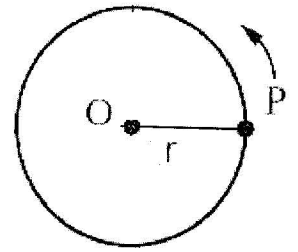
$$f = \frac{K_f - K_i}{K_i} = -\frac{M}{m + M} = -\frac{0.8}{0.804} = -0.995$$

$\therefore f = 99.5\%$ lost due to collision.

$$f = 99.5\%$$

QUESTION 5 (20 %)

A wheel starts from rest and rotates with a constant angular acceleration to reach an angular speed of 12 rad/s in 3 s . The wheel has a point P on its rim. The radius of the wheel from the point P is 0.5 m . Find



(a) the magnitude of the angular acceleration of the wheel,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{12 - 0}{3} = 4 \text{ rad/s}^2$$

$$\alpha = 4 \text{ rad/s}^2$$

(b) the angle in radians through which it rotates in this time,

$$\begin{aligned} \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ &= 0 + 0 + \frac{1}{2} 4 (3)^2 \\ &= 18 \text{ rad} \end{aligned}$$

$$\theta = 18 \text{ rad}$$

(c) the tangential speed of the point P at $t = 3 \text{ s}$,

$$v = \omega r = (12)(0.5) = 6 \text{ m/s}$$

$$v = 6 \text{ m/s}$$

(d) the tangential acceleration of the point P at $t = 3 \text{ s}$ and

$$a_t = \alpha r = (4)(0.5) = 2 \text{ m/s}^2$$

$$a_t = 2 \text{ m/s}^2$$

(e) the radial acceleration of the point P at $t = 3 \text{ s}$.

$$a_r = \omega^2 r = (12)^2 (0.5) = 72 \text{ m/s}^2$$

$$a_r = 72 \text{ m/s}^2$$