



Date: 07/11/2019 Time: 10:30 Duration: 90 min.

DEPARTMENT : CE  MME  IE  ME  TE

Name \_\_\_\_\_ Surname \_\_\_\_\_ Student No \_\_\_\_\_ Signature \_\_\_\_\_

SOLUTIONS

Ques.	Mark
1	
2	
3	
4	
5	
Total	

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants:  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3.141593$
- Conversions:  $1 \text{ g} = 10^{-3} \text{ kg}$ ,  $1 \text{ cm} = 10^{-2} \text{ m}$ ,  $1 \text{ km} = 10^3 \text{ m}$ ,  $1 \text{ h} = 3600 \text{ s}$ ,  $1 \text{ min} = 60 \text{ s}$ ,  $1 \text{ rev} = 2\pi \text{ rad}$ .

### QUESTION 1 (20 %)

A particle moves on the x-y plane. The position vector of the particle is given by

$$\vec{r}(t) = (t-10)\hat{i} + 2t\hat{j} \text{ (m)}$$

where  $t$  is the time in seconds.

(a) Find the displacement vector of the particle between  $t=0$  and  $t=5$  s.

$$\vec{r}(5) = (5-10)\hat{i} + 2(5)\hat{j} = -5\hat{i} + 10\hat{j} \text{ (m)}$$

$$\vec{r}(0) = (0-10)\hat{i} + 2(0)\hat{j} = -10\hat{i} \text{ (m)}$$

$$\Delta\vec{r} = \vec{r}(5) - \vec{r}(0) = -5\hat{i} + 10\hat{j} - (-10\hat{i}) \Rightarrow$$

$$\Delta\vec{r} = 5\hat{i} + 10\hat{j} \text{ (m)}$$

(b) Find the instantaneous velocity vector,  $\vec{v}$ , of the particle at time  $t=5$  s.

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt} \{ (t-10)\hat{i} + 2t\hat{j} \}$$

$$= \hat{i} + 2\hat{j} \text{ (m/s)}$$

$$\vec{v}(5) = \hat{i} + 2\hat{j} \text{ (m/s)}$$

$$\vec{v} = \hat{i} + 2\hat{j} \text{ (m/s)}$$

(c) What is the position vector of particle at a time corresponding to  $\vec{r} \cdot \vec{v} = 0$ ?

$$\vec{r} \cdot \vec{v} = [(t-10)\hat{i} + 2t\hat{j}] \cdot [\hat{i} + 2\hat{j}] = 0$$

$$= t-10 + 4t = 0 \Rightarrow t = 2 \text{ s.}$$

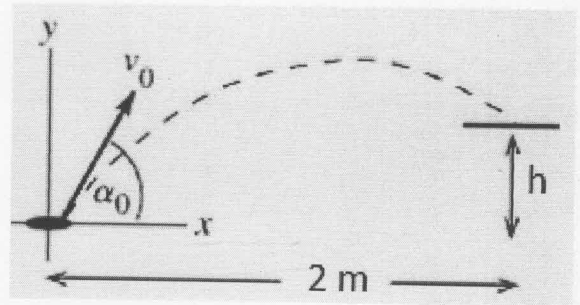
$$\vec{r}(2) = (2-10)\hat{i} + 2(2)\hat{j}$$

$$= -8\hat{i} + 4\hat{j} \text{ (m)}$$

$$\vec{r} = -8\hat{i} + 4\hat{j} \text{ (m)}$$

## QUESTION 2 (20 %)

An object is thrown from ground into the air with an initial speed  $V_0 = 5 \text{ m/s}$  and an angle of  $\alpha_0 = 60^\circ$ . If its height is  $h$  when it reaches a horizontal distance of  $2 \text{ m}$ , find



(a) the components of the initial velocity of object,

$$\begin{aligned} V_{0x} &= V_0 \cos \alpha_0 \\ &= 5 \cos 60^\circ \\ &= 2.5 \text{ m/s} \end{aligned} \quad \left| \quad \begin{aligned} V_{0y} &= V_0 \sin \alpha_0 \\ &= 5 \sin 60^\circ \\ &= 4.33 \text{ m/s} \end{aligned} \right.$$

$$V_{0x} = 2.50 \text{ m/s}$$

$$V_{0y} = 4.33 \text{ m/s}$$

(b) the time to reach this height,

$$x = v_{0x} t \rightarrow t = \frac{x}{v_{0x}} = \frac{2}{2.5} = 0.8 \text{ s}$$

$$t = 0.8 \text{ s}$$

(c) the height  $h$ , and

$$\begin{aligned} y = h &= v_{0y} t - \frac{1}{2} g t^2 \\ &= (4.33)(0.8) - \frac{1}{2} (9.8)(0.8)^2 \\ &= 0.34 \text{ m} \end{aligned}$$

$$h = 0.34 \text{ m}$$

(d) the components of velocity at this height.

$$\begin{aligned} V_x &= V_{0x} = 2.5 \text{ m/s} \\ V_y &= V_{0y} - g t \\ &= 4.33 - (9.8)(0.8) \\ &= -3.5 \text{ m/s} \end{aligned}$$



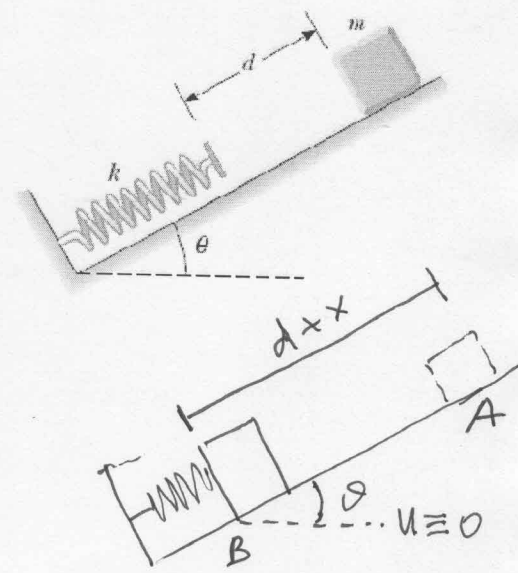
$$V_x = 2.5 \text{ m/s}$$

$$V_y = -3.5 \text{ m/s}$$

**QUESTION 3 (20 %)**

An object of mass  $m=3 \text{ kg}$  starts from rest and slides a distance  $d$  down a frictionless incline of angle  $\theta=30^\circ$ . While sliding, it contacts an unstressed spring of negligible mass as shown in the figure. The object slides an additional distance  $x=0.2 \text{ m}$  as it is brought momentarily to rest by compression of the spring (of force constant  $k=400 \text{ N/m}$ ). Using the conservation of mechanical energy, find the initial separation  $d$  between object and spring.

**Hint:** Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest.



we can choose  $U_g = 0$  in which the object comes to rest. since there is no friction,  $E_A = E_B$ .

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$0 + mg(d+x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2$$

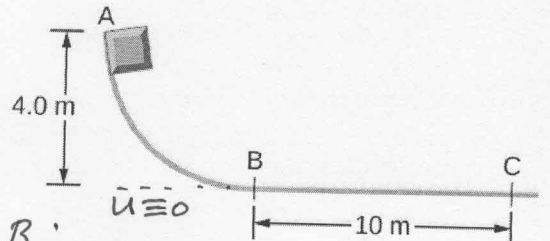
$$\text{or } d = \frac{kx^2}{2mg\sin\theta} - x = 0.34 \text{ m}$$

$$d = 0.34 \text{ m}$$

**QUESTION 4 (20 %)**

A small block of mass  $240 \text{ g}$  starts at rest at point A, slides to point B where its speed is  $v_B = 7 \text{ m/s}$ . Then it slides along the horizontal surface a distance of  $10 \text{ m}$  before coming to rest at point C.

(a) What is the work done by friction along the curved surface?



- $m = 0.24 \text{ kg}$
- $h = 4 \text{ m}$
- $v_B = 7 \text{ m/s}$
- $d = 10 \text{ m}$

Work done by friction along curve A-B:

$$\begin{aligned} W_f &= \Delta E \\ &= E_B - E_A \\ &= \frac{1}{2}mv_B^2 - mgh \\ &= \frac{1}{2}(0.24)(7)^2 - (0.24)(9.8)(4) \\ &= -3.5 \text{ J} \end{aligned}$$

$$W_f = -3.5 \text{ J}$$

(b) What is the coefficient of kinetic friction along the horizontal surface?

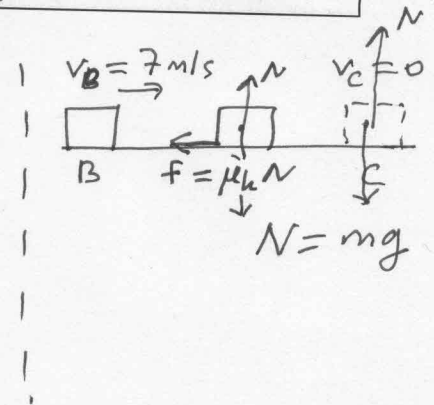
method I

$$\begin{aligned} W_f &= E_C - E_B \\ -fd &= 0 - \frac{1}{2}mv_B^2 \\ -\mu_k mgd &= -\frac{1}{2}mv_B^2 \\ \mu_k &= \frac{v_B^2}{2gd} \\ &= \frac{7^2}{(2)(9.8)(10)} = 0.25 \end{aligned}$$

method II

eqn. of motion:

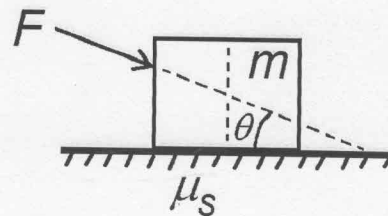
$$\begin{aligned} v_C^2 &= v_B^2 - 2ad \\ a &= v_B^2 / 2d \\ F_{\text{net}} &= ma \\ f &= ma \\ \mu_k mg &= m \left( \frac{v_B^2}{2d} \right) \\ \mu_k &= \frac{v_B^2}{2gd} = 0.25 \end{aligned}$$



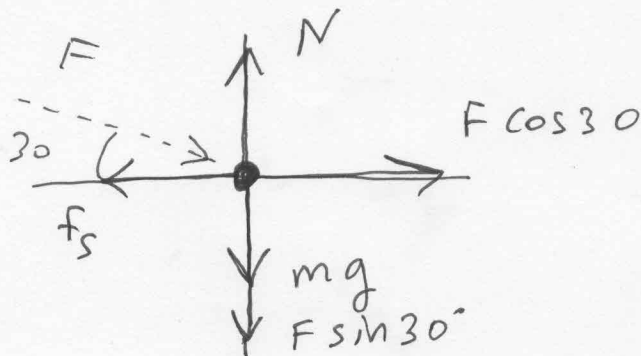
$$\mu_k = 0.25$$

**QUESTION 5 (20 %)**

A block of mass  $m = 12 \text{ kg}$  is pushed with a force  $F$  at an angle of  $\theta = 30^\circ$  on a horizontal floor that has a coefficient of static friction  $\mu_s = 0.3$  as shown in Figure.



(a) Show all forces acting on the object (Namely, draw the free-body diagram of the system).



$$m = 12 \text{ kg}$$

$$\theta = 30^\circ$$

$$\mu_s = 0.3$$

$$f_s = \mu_s N$$

$$= \mu_s (mg + F \sin \theta)$$

(b) What minimum value of force,  $F$  is required to move the block?

Newton's 2<sup>nd</sup> law:

$$F \cos 30 - f_s = ma = 0 \quad \leftarrow \text{since the block is at rest}$$

$$F \cos 30 - \mu_s (mg + F \sin \theta) = 0$$

Solving for  $F$ :

$$F = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta} = \frac{(0.3)(12)(9.8)}{\cos 30 - (0.3) \sin 30}$$

$$= 49.3 \text{ N}$$

This is the minimum value.

$$F = 49.3 \text{ N}$$

$\therefore F \geq 49.3 \text{ N}$  to move the block.