



Date: 03/01/2019 Time: 10:30 Duration: 100 min.

EDUCATION :  1<sup>st</sup> Ed.  2<sup>nd</sup> Ed.  
DEPARTMENT :  CE  MME  IE  ME  TE

Ques.	Mark
1	
2	
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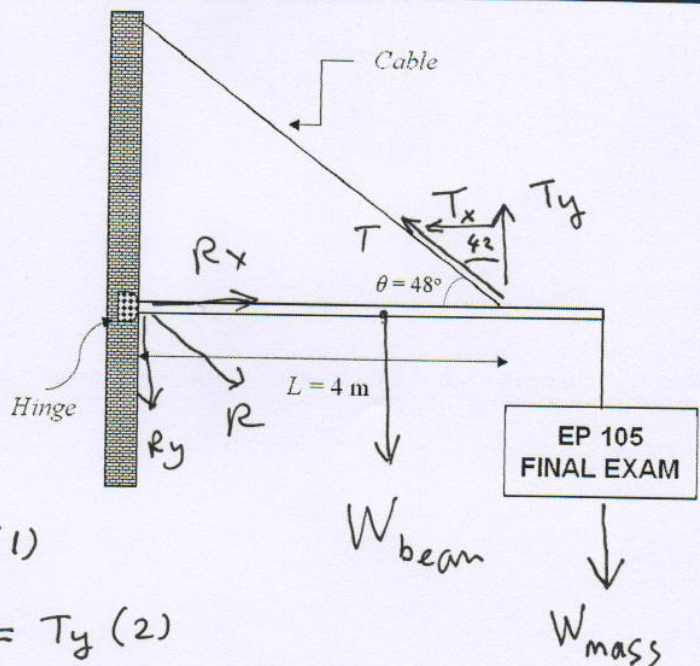
Name	Surname	Student No	Signature
	SOLUTIONS		

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants:  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3.141593$
- $1 \text{ mm} = 10^{-3} \text{ m}$ ,  $1 \text{ cm} = 10^{-2} \text{ m}$ ,  $1 \text{ nm} = 10^{-9} \text{ m}$ ,  $1 \text{ km} = 10^3 \text{ m}$ ,  $1 \text{ h} = 3600 \text{ s}$ ,  $1 \text{ min} = 60 \text{ s}$ ,  $1 \text{ rev} = 2\pi \text{ rad}$ .

### QUESTION 1 (20 %)

A 6 m uniform beam of mass 32 kg is suspended horizontally by a hinged end and a cable. A 93 kg plate is connected to one end of the beam as shown in the figure right.

Find the magnitude of the tension in the cable and the magnitude of the reaction force that the hinge exerts on the beam.



$$W_{\text{beam}} = (32)(9.8) = 314 \text{ N}$$

$$W_{\text{mass}} = (93)(9.8) = 911 \text{ N}$$

$$\sum F_x = 0 \rightarrow R_x - T_x = 0 \quad (1)$$

$$\sum F_y = 0 \rightarrow R_y + 314 + 911 = T_y \quad (2)$$

$$\sum \tau_o = 0 \rightarrow (314)(3) + (911)(6) = 4T_y \Rightarrow T_y = 1600 \text{ N}$$

$$\tan 42^\circ = \frac{T_x}{T_y} \rightarrow T_x = (1600) \tan 42^\circ = 1440 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = 2152 \text{ N.}$$

From eqn(1) :  $R_x = T_x = 1440 \text{ N}$

From eqn(2) :  $R_y = 1600 - 1225 = 375 \text{ N}$

$$T = 2152 \text{ N}$$

$$R = 1488 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = 1488 \text{ N}$$

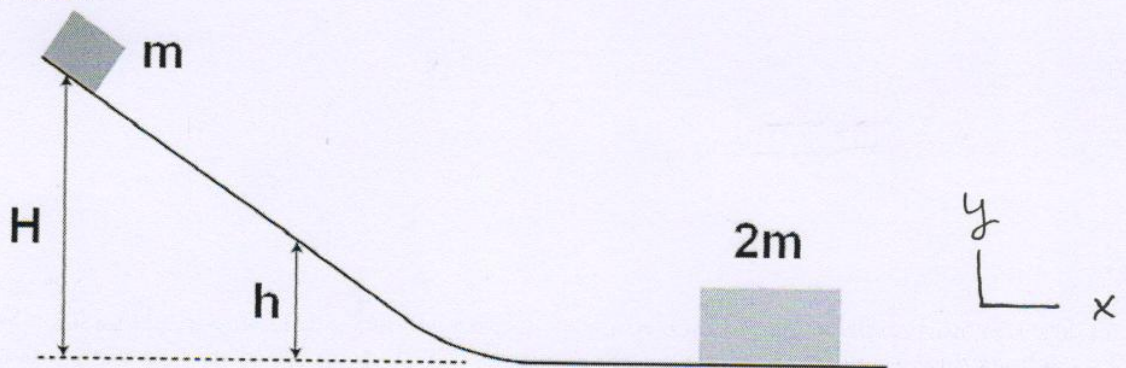
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**QUESTION 2 (20 %)**

A block of mass  $m = 2.0$  kg slides down the frictionless ramp shown below. The block starts from rest at a height  $H = 16.2$  m above the bottom of the ramp. At the bottom, it makes an elastic collision with another block of mass  $2m$  at rest.



(a) Calculate the speed of each block after the collision.

This is an elastic collision in one dimension.

Conservation of energy for mass  $m$ :

$$mgH = \frac{1}{2} m v_{1i}^2$$

$$v_{1i} = \sqrt{2gH}$$

$$= 17.8 \text{ m/s}$$

Final velocities after collision:

- $v_{1f} = \frac{m-2m}{m+2m} v_{1i} = -6.0 \text{ m/s}$
- $v_{2f} = \frac{2m}{m+2m} v_{1i} = +12 \text{ m/s}$

5

$$v_{1f} = -6 \text{ m/s}$$

5

$$v_{2f} = 12 \text{ m/s}$$

(b) If lighter block ( $m = 2$  kg) will travel back up the incline to a height  $h$  after the collision, then calculate the ratio of the two heights, namely  $h/H$ ?

Conservation of energy for  $m$  after collision:

$$\frac{1}{2} m v_{1f}^2 = mgh$$

or

$$h = \frac{v_{1f}^2}{2g} = \frac{6^2}{2 \times 9.8} = 1.84 \text{ m}$$

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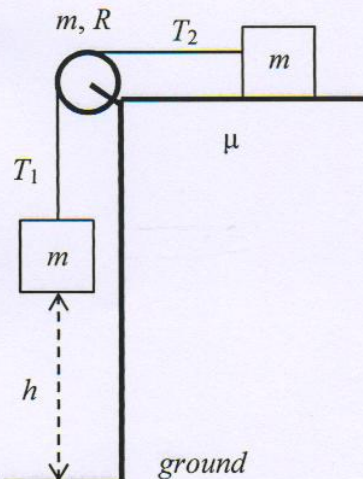
$$\frac{h}{H} = \frac{1.84}{16.2} = 0.11 = \frac{1}{9}$$

$$h/H = 1/9$$



**QUESTION 3 (20 %)**

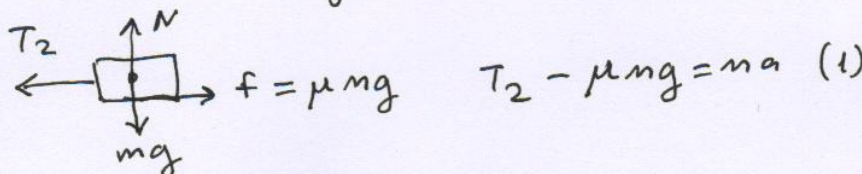
Two identical blocks, each of mass  $m = 2 \text{ kg}$ , are connected by a light string, over a frictionless pulley of radius  $R$  and mass  $m$ . The coefficient of kinetic friction between plane and sliding block is  $\mu = 0.1$ . The lower block falls down from a height  $h = 0.8 \text{ m}$  before it hits the ground. Assume that the system is released at time  $t = 0$ . (The rotational inertia of the pulley is  $I = mR^2/2$ )



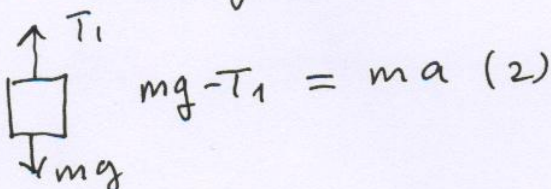
(a) What is the acceleration of two blocks?

Newton's 2<sup>nd</sup> law

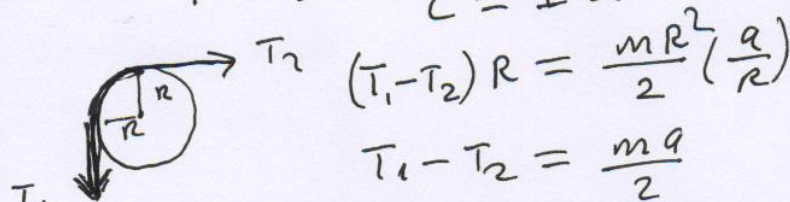
• for sliding block:



• for falling block:



• for pulley:



adding (1), (2) and (2)

$$mg - \mu mg = ma + ma + \frac{mg}{2}$$

$$g(1 - \mu) = \frac{5a}{2}$$

$$\text{or } a = \frac{2g(1 - \mu)}{5}$$

$$= 3.53 \text{ m/s}^2 \quad (7)$$

$$a = 3.53 \text{ m/s}^2$$

(b) What is the tension in the lower and upper sections of the string?

from eqn(1):  $T_2 = ma + \mu mg = 2 \times 3.53 + 0.1 \times 2 \times 9.8 = 9.02 \text{ N}$

from eqn(2):  $T_1 = mg - ma = 12.54 \text{ N}$

$$T_1 = 12.54 \text{ N} \quad (4)$$

$$T_2 = 9.02 \text{ N} \quad (4)$$

(c) What is the speed of the block when it hits the ground?

one can use equation of motion with constant acc.

$$v^2 = v_0^2 + 2ah$$

$$= 0 + (2)(3.53)(0.8)$$

$$= 5.64 \text{ m}^2/\text{s}^2$$

$$\Rightarrow v = 2.38 \text{ m/s}$$

(5)

$$v = 2.38 \text{ m/s}$$



**QUESTION 4 (20 %)**

In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression  $x = (60 \text{ cm}) \cos(\pi t / 8 - \pi / 2)$  where  $x$  is in centimeters and  $t$  is in seconds. The spring constant is  $100 \text{ N/cm}$ .

(a) What is the period of the oscillation?

$$x = A \cos(\omega t + \phi)$$

$$x = 60 \cos\left(\frac{\pi}{8}t - \frac{\pi}{2}\right)$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/8} = 16 \text{ s}$$

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$$T = 16 \text{ s}$$

(b) Find the kinetic energy of the particle at time  $t = 2 \text{ sec}$ .

$$v = \frac{dx}{dt} = \frac{d}{dt} \left\{ 60 \cos\left(\frac{\pi}{8}t - \frac{\pi}{2}\right) \right\} = (60 \text{ cm}) \left(-\frac{\pi}{8}\right) \sin\left(\frac{\pi}{8}t - \frac{\pi}{2}\right)$$

$$v(2) = -16.67 \text{ cm/s} \cdot \omega^2 = k/m \Rightarrow m = \frac{1}{(\pi/8)^2} = 6.48 \text{ kg}$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} (6.48) (-0.167)^2 = 0.09 \text{ J}$$

$$K = 0.09 \text{ J}$$

(c) Find the acceleration of the motion at time  $t = 2 \text{ sec}$ .

$$a = \frac{dv}{dt} = \frac{d}{dt} \left\{ 60 \text{ cm} \left(-\frac{\pi}{8}\right) \sin\left(\frac{\pi}{8}t - \frac{\pi}{2}\right) \right\}$$

$$= -60 \text{ cm} \left(\frac{\pi}{8}\right)^2 \cos\left(\frac{\pi}{8}t - \frac{\pi}{2}\right)$$

$$a(2) = -6.5 \text{ cm/s}^2$$

8

8

$$a = -6.5 \text{ cm/s}^2$$

**QUESTION 5 (20 %)**

Adem Ljajić is a professional football player who plays as an attacking midfielder for Beşiktaş FC. The position vector of Adem Ljajić varies in time according to the expression  $\mathbf{r} = (0.01t^3 \mathbf{i} - 0.12t^2 \mathbf{j}) \text{ m}$  where  $t$  is time in second.

(a) Determine the magnitude of his velocity at  $t = 8 \text{ s}$ .

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} \left\{ 0.01t^3 \hat{i} - 0.12t^2 \hat{j} \right\}$$

$$= 0.03t^2 \hat{i} - 0.24t \hat{j} \text{ (m/s)}$$

$$\text{at } t=8 \rightarrow \vec{v} = 1.92 \hat{i} - 1.92 \hat{j} \text{ (m/s)}$$

$$|\vec{v}| = 2.715 \text{ m/s}$$

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$$v = 2.715 \text{ m/s}$$

(b) If he has a mass of  $74 \text{ kg}$ , find the magnitude of his momentum at  $t = 8 \text{ s}$ .

momentum:

$$\vec{p} = m\vec{v} = (74)(1.92 \hat{i} - 1.92 \hat{j}) = 142(\hat{i} - \hat{j}) \text{ kg}\cdot\text{m/s}$$

$$|\vec{p}| = m|\vec{v}| = 74 \times 2.715 = 200.91 \text{ kg}\cdot\text{m/s}^2 \approx 201 \text{ kg}\cdot\text{m/s}^2$$

$$p = 201 \text{ kg}\cdot\text{m/s}$$

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