



Date: 11/12/2018 Time: 08:30 Duration: 100 min.

Ques.	Mark
1	
2	
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4	
5	
Total	

EDUCATION :  1<sup>st</sup> Ed.  2<sup>nd</sup> Ed.  
DEPARTMENT :  CE  MME  IE  ME  TE

Name	Surname	Student No	Signature
	SOLUTIONS		

- Cheating is a serious offence and may lead to your dismissal from the university.
- Ignore air resistance in all problems unless otherwise stated.
- Write clearly your solutions steps to the space provided and results to the boxes.
- Constants:  $g = 9.8 \text{ m/s}^2$ ,  $\pi = 3.141593$
- Conversions:  $1 \text{ mm} = 10^{-3} \text{ m}$ ,  $1 \text{ cm} = 10^{-2} \text{ m}$ ,  $1 \text{ nm} = 10^{-9} \text{ m}$ ,  $1 \text{ km} = 10^3 \text{ m}$ ,  $1 \text{ h} = 3600 \text{ s}$ ,  $1 \text{ min} = 60 \text{ s}$ ,  $1 \text{ rev} = 2\pi \text{ rad}$ .

**QUESTION 1 (20 %)**

Figure shows a uniform disk of radius  $R = 0.1 \text{ m}$  mounted on a fixed horizontal axle. A block with mass  $m = 2 \text{ kg}$  hangs from a massless cord that is wrapped around the rim of the disk. The cord does not slip and there is no friction at the axle. The mass  $m$  falls down at a constant acceleration of  $a = 5 \text{ m/s}^2$ .

(a) Calculate the rotational inertia of the disk about the axle.

Newton's 2<sup>nd</sup> law:

$$mg - T = ma \quad (\text{for mass})$$

$$\tau = I\alpha \quad (\text{for disk})$$

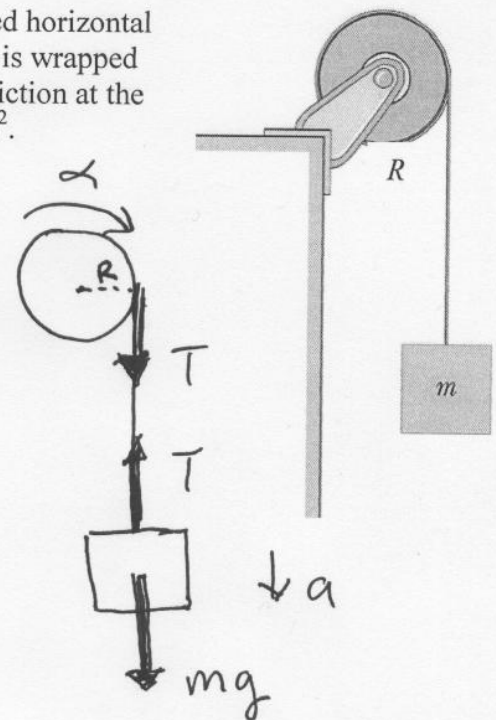
$$TR = I \frac{a}{R}$$

$$T = mg - ma$$

$$I = \frac{TR^2}{a} = \frac{(mg - ma)R^2}{a}$$

$$= \frac{(2 \times 9.8 - 2 \times 5)(0.1)^2}{5}$$

$$= 0.0192 \text{ kg} \cdot \text{m}^2$$



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$$I = 0.0192 \text{ kg} \cdot \text{m}^2$$

(b) Find the tension in the cord while the mass is falling down.

$$T = mg - ma = 9.6 \text{ N}$$

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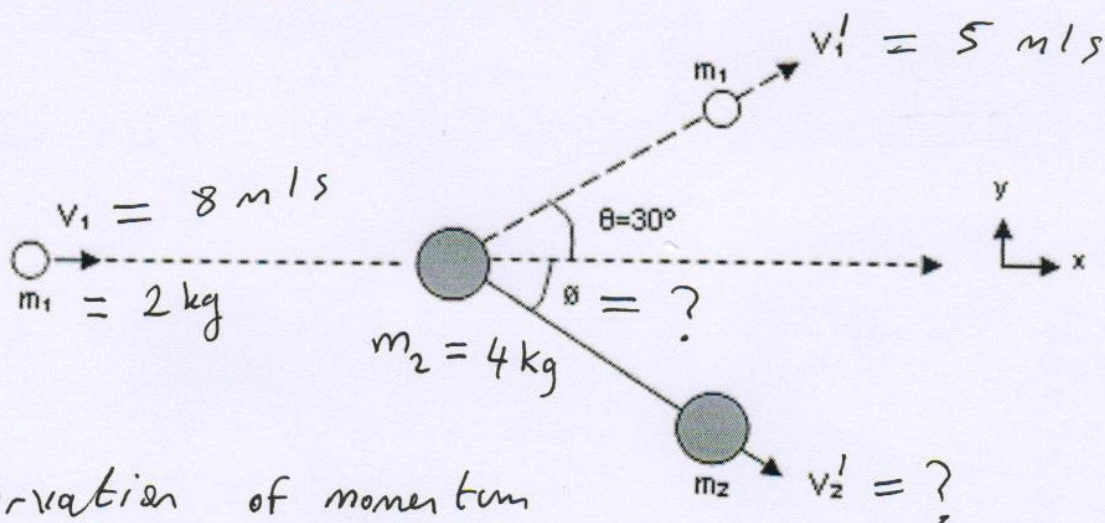
or

$$T = \frac{Ia}{R^2} = 9.6 \text{ N}$$

$$T = 9.6 \text{ N}$$

**QUESTION 2 (20 %)**

A ball of mass  $m_1 = 2 \text{ kg}$  and speed  $v_1 = 8 \text{ m/s}$  makes a collision with a target particle ( $m_2 = 4 \text{ kg}$ ) at rest. After the collision the ball  $m_1$  is scattered with  $v_1' = 5 \text{ m/s}$  and  $\theta = 30^\circ$ , as shown. Find the direction ( $\phi$ ) and the velocity of the ball  $m_2$  after the collision.



Conservation of momentum

• in x-direction:  $m_1 v_1 + 0 = m_1 v_1' \cos \theta + m_2 v_2' \cos \phi$   
 $(2)(8) = (2)(5) \cos 30 + 4 v_2' \cos \phi$   
 $1.835 = v_2' \cos \phi \quad (1) \quad 5$

• in y-direction:  $0 = m_1 v_1' \sin \theta - m_2 v_2' \sin \phi$   
 $0 = (2)(5) \sin 30 - 4 v_2' \sin \phi$   
 $1.250 = v_2' \sin \phi \quad (2) \quad 5$

If you divide Eq(2) by Eq(1)

$$\tan \phi = \frac{1.250}{1.835} = 0.681 \rightarrow \phi = 34.3^\circ$$

or  $\phi = 0.6 \text{ rad.}$

from Eq(1)

$$v_2' = \frac{1.835}{\cos \phi} = \frac{1.835}{\cos(34.3)} = 2.2 \text{ m/s} \quad 5$$

$$\phi = 34.3^\circ$$

$$v_2' = 2.2 \text{ m/s}$$

**QUESTION 3 (20 %)**

A bicycle wheel has an initial angular velocity of 1.5 rad/s. Angular acceleration of the wheel is constant and equal to 0.3 rad/s<sup>2</sup>.

(a) What is the angular velocity (in rad/s) of the wheel at time  $t = 2.5$  s?

$$\begin{aligned} \omega &= \omega_0 + \alpha t \\ &= 1.5 + (0.3)(2.5) \\ &= 2.25 \text{ rad/s} \end{aligned}$$

$\omega = 2.25 \text{ rad/s}$

(b) Through what angle (in rad) has the wheel turned between  $t = 0$  and  $t = 2.5$  s?

$$\begin{aligned} \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\ &= 0 + (1.5)(2.5) + \frac{1}{2} (0.3) (2.5)^2 \\ &= 4.69 \text{ rad} \end{aligned}$$

$\theta = 4.69 \text{ rad}$

**QUESTION 4 (20 %)**

A water molecule consists of an oxygen atom with two hydrogen atoms bound to it as shown in Figure. The angle between the two bonds is 106°. If the bonds are  $d = 0.1$  nm long, where is the center of mass of the molecule? Masses of Hydrogen and Oxygen are  $1.674 \times 10^{-27}$  kg and  $26.567 \times 10^{-27}$  kg respectively.

Particle	$m \times 10^{-27}$ kg	$x$ (nm)	$y$ (nm)
1	26.567	0.000	0.000
2	1.674	0.006	0.008
3	1.674	0.006	-0.008

Sum = 29.915

$$\begin{aligned} x_{cm} &= \frac{\sum m_i x_i}{\sum m_i} \\ &= \frac{0 + (1.674)(0.006) + (1.674)(0.006)}{29.915} \end{aligned}$$

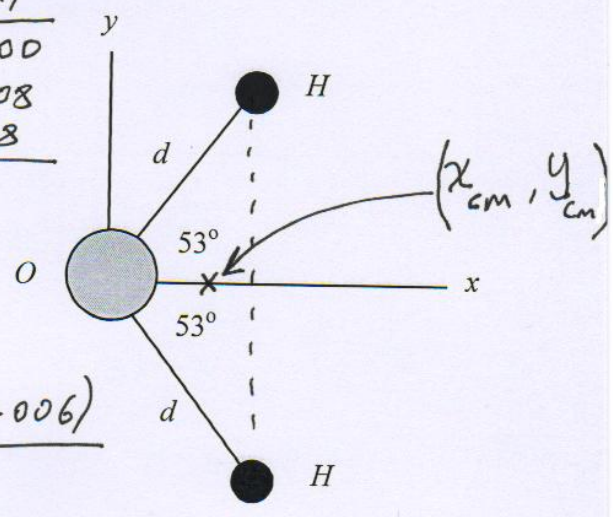
= 0.0067 nm 15

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{0 + (1.674)(0.008) + (1.674)(-0.008)}{29.915} = 0$$

$x_{cm} = 0.0067 \text{ nm}$

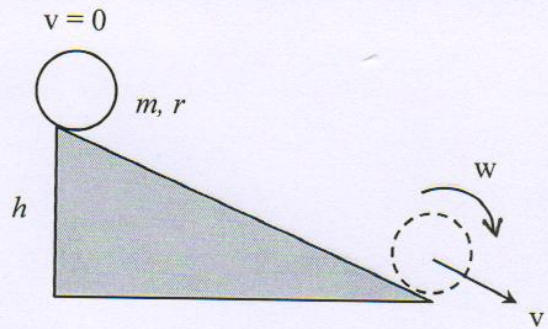
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$y_{cm} = 0.0000$



**QUESTION 5 (20 %)**

A 5.2 kg hoop (ring) which has a radius of  $r = 13$  cm is released at the top of an inclined surface of  $h = 3.2$  m above the ground. Assume that the hoop rolls down without slipping. [ $I = mr^2$ ]



- (a) Calculate the center of mass velocity of the hoop at the bottom of the incline. Use Conservation of Energy principle.

$$\text{Gravitational Potential Energy} = \text{Translational K.E.} + \text{Rotational K.E.}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\frac{v^2}{r^2}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$gh = v^2$$

$$v = \sqrt{gh} = \sqrt{(9.8)(3.2)}$$

$$= 5.6 \text{ m/s}$$

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$$v = 5.6 \text{ m/s}$$

- (b) Calculate the angular velocity about center of mass of the hoop at the bottom of the incline.

$$\omega = \frac{v}{r} = \frac{5.6 \text{ m/s}}{0.13 \text{ m}} = 43.1 \text{ rad/s}$$

5

$$\omega = 43.1 \text{ rad/s}$$