



Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
OUT OF	100

Student No	Name	Surname	Dep.	Signature

**\*\*\* SOLVE ONLY 5 OUT OF 6 QUESTIONS \*\*\***

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants:  $g = 9.8 \text{ m/s}^2$

### QUESTION 1 (20 %)

An object is suspended by a cord as shown in Fig I and the tension in the cord is  $T_1 = 10 \text{ N}$ . The object is then immersed to the water as shown in Fig II. In this case the tension in the cord is  $T_2 = 6 \text{ N}$ . Determine the mass density of the object.

(Density of the water is given as:  $\rho_w = 1000 \text{ kg/m}^3$ )

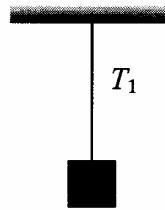


Fig I

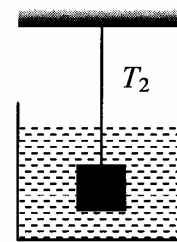
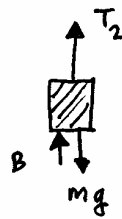


Fig II

*Equilibrium conditions for each case:*



$$T_1 = mg \quad (1)$$



$$B = mg - T_2$$

$$\rho_w V g = T_1 - T_2 \quad (2)$$

*Density of the object:*

$$\rho = \frac{m}{V} = \frac{mg}{Vg} = \frac{T_1}{(T_1 - T_2)/\rho_w} = \left( \frac{T_1}{T_1 - T_2} \right) \rho_w$$

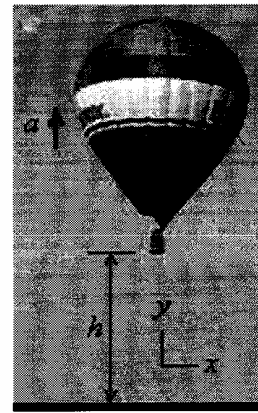
$$= \left( \frac{10}{10 - 6} \right) (1000)$$

$$= 2500 \text{ kg/m}^3$$

$\rho = 2500 \text{ kg/m}^3$
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**QUESTION 2 (20 %)**

A hot-air balloon which is initially at rest is rising vertically at a constant acceleration of  $a = 2 \text{ m/s}^2$ . When the balloon is  $h = 225 \text{ m}$  above the ground, an object of negligible mass is dropped (released) from the balloon at time  $t = 0$ .



(a) Find the velocity of the balloon at height  $h = 225 \text{ m}$

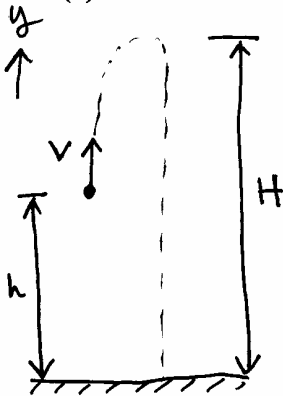
$$v^2 = v_0^2 + 2ah \rightarrow v = \sqrt{2ah}$$

$$= \sqrt{(2)(2)(225)}$$

$$= 30 \text{ m/s}$$

$v = 30 \text{ m/s}$

(b) Find the maximum height  $H$  from the ground reached by the object



$$v'^2 = v^2 - 2g(H-h)$$

$$0 = v^2 - 2g(H-h)$$

or

$$H = \frac{v^2}{2g} + h = \frac{30^2}{2 \times 9.8} + 225 = 270.9 \text{ m}$$

$H = 270.9 \text{ m}$

(c) Find the time  $t$  at which the object hits the ground

$$y_f = y_0 + vt - \frac{1}{2}gt^2$$

$$t_{1,2} = \frac{30 \pm \sqrt{30^2 - 4(4.9)(-225)}}{(2)(4.9)}$$

$$0 = h + vt - \frac{1}{2}gt^2$$

or

$$4.9t^2 - 30t - 225 = 0$$

$$\therefore t_1 = 10.5 \text{ s} \quad \checkmark$$

$$t_2 = -4.45 \text{ s} \quad \times$$

$t = 10.5 \text{ s}$

(d) Find the height from the ground of the balloon when the object hits the ground

$$h' = h + vt + \frac{1}{2}at^2$$

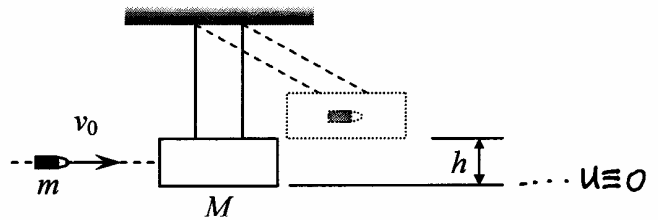
$$= 225 + 30(10.5) + \frac{1}{2}(2)(10.5)^2$$

$$= 650.25 \text{ m}$$

$h' = 650.25 \text{ m}$

### QUESTION 3 (20 %)

The ballistic pendulum (as in the figure) is a system used to measure the speed of a bullet. A bullet of mass  $m = 5 \text{ g}$  is fired into a large wooden block of  $M = 1 \text{ kg}$  suspended from light wires. The bullet embeds in the block, and the entire system swings through a height  $h = 5 \text{ cm}$ .



(a) Find the initial speed ( $v_0$ ) of the bullet

Conservation of momentum:

$$m v_0 = (m+M) v \quad (1)$$

where  $v$  is the common velocity of the system  $m+M$  after the collision.

Mechanical energy is conserved after the collision.

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} (m+M) v^2 = (m+M) g h$$

$$v^2 = 2 g h \quad (2)$$

Combining Eqn (1) and (2) and solving for  $v_0$

$$\begin{aligned} v_0 &= \left( \frac{m+M}{m} \right) \sqrt{2 g h} \\ &= \left( \frac{5 \times 10^{-3} + 1}{5 \times 10^{-3}} \right) \sqrt{(2)(9.8)(5 \times 10^{-2})} \\ &= 199 \text{ m/s} \end{aligned}$$

$$v_0 = 199 \text{ m/s}$$

(b) Find the loss in mechanical energy ( $\Delta E$ ) due to the collision

Mechanical energy before collision:

$$E_1 = \frac{1}{2} m v_0^2 = \frac{1}{2} (5 \times 10^{-3}) (199)^2 = 99 \text{ J}$$

Mechanical energy after collision:

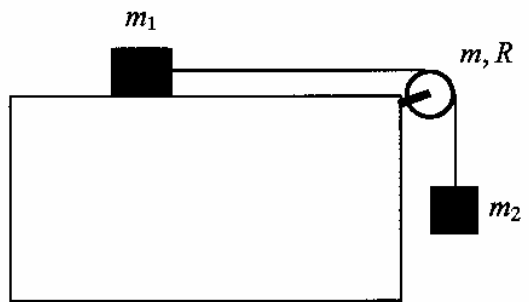
$$\begin{aligned} E_2 &= \frac{1}{2} (m+M) v^2 = (m+M) g h \\ &= (5 \times 10^{-3} + 1) (9.8) (5 \times 10^{-2}) \\ &= 0.49 \text{ J} \end{aligned}$$

$$\therefore \Delta E = E_2 - E_1 = -98.5 \text{ J}$$

$$\Delta E = -98.5 \text{ J}$$

**QUESTION 4 (20 %)**

Two blocks,  $m_1 = 2 \text{ kg}$  and  $m_2 = 5 \text{ kg}$ , are connected by a light strings that passes over a pulley of radius  $R = 10 \text{ cm}$  and mass  $m = 1 \text{ kg}$  as seen in the figure. The coefficient of kinetic friction between the mass  $m_1$  and surface is  $\mu = 0.2$ . (Moment of inertia of the pulley about its center of mass is  $I = mR^2/2$ )



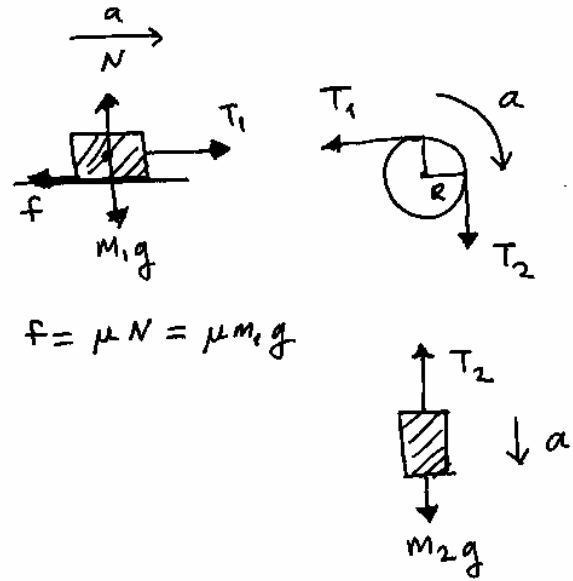
(a) Calculate the common acceleration of the blocks

Newton's 2<sup>nd</sup> Law

for block 1:  $T_1 - f = m_1 a$   
 $T_1 - \mu m_1 g = m_1 a \quad (1)$

for block 2:  $m_2 g - T_2 = m_2 a \quad (2)$

for pulley:  $\tau = I \alpha$   
 $(T_2 - T_1)R = \left(\frac{mR^2}{2}\right) \frac{a}{R}$   
 $T_2 - T_1 = \frac{ma}{2} \quad (3)$



$f = \mu N = \mu m_1 g$

Adding Eqn (1), (2) and (3) gives:

$a = \left(\frac{m_2 - \mu m_1}{m_2 + m_1 + m/2}\right) g = \left(\frac{5 - 0.2 \times 2}{5 + 2 + 1/2}\right) 9.8 = 6 \text{ m/s}^2$

$a = 6 \text{ m/s}^2$

(b) Calculate tensions in the string

Using Eqn (1):  $T_1 = m_1 (a + \mu g) = 2(6 + 0.2 \times 9.8) \approx 16 \text{ N}$

Using Eqn (2):  $T_2 = m_2 (g - a) = 5(9.8 - 6) = 19 \text{ N}$

$T_1 = 16 \text{ N}$   
 $T_2 = 19 \text{ N}$

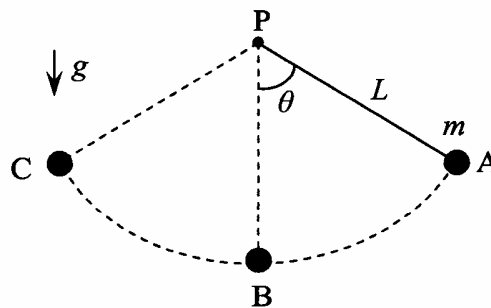
(c) Calculate angular acceleration of the pulley

$\alpha = \frac{a}{R} = \frac{6}{0.1} = 60 \text{ rad/s}^2$

$\alpha = 60 \text{ rad/s}^2$

**QUESTION 5 (20 %)**

A pendulum consists of a ball of mass  $m = 2 \text{ kg}$  attached to a light cord of length  $L = 3.2 \text{ m}$  as shown in Figure. The ball is released from rest at point A when the cord makes an angle  $\theta = 60^\circ$  with the vertical. The ball passes through its lowest point B and stops at point C. Ignore all possible frictions in the system.

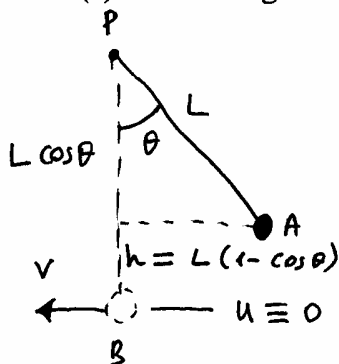


(a) Find the rotational inertia of the ball about point P

$$I = mL^2 = 2 \times (3.2)^2 = 20.5 \text{ kg}\cdot\text{m}^2$$

$$I = 20.5 \text{ kg}\cdot\text{m}^2$$

(b) Find the angular speed of the ball (about point P) at point B



Conservation of Energy:

$$\begin{aligned} K_A + U_A &= K_B + U_B \\ mgh &= \frac{1}{2}mv^2 \\ v &= \sqrt{2gh} \\ &= \sqrt{2gL(1 - \cos\theta)} \\ &= \sqrt{(2)(9.8)(3.2)(1 - \cos 60^\circ)} \\ &= 5.6 \text{ m/s} \end{aligned}$$

Angular Velocity:

$$\begin{aligned} \omega &= \frac{v}{L} \\ &= \frac{5.6}{3.2} \\ &= 1.75 \text{ rad/s} \end{aligned}$$

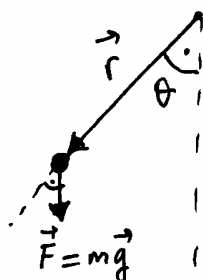
$$\omega = 1.75 \text{ rad/s}$$

(c) Find the magnitude of the angular momentum (about point P) of the ball at point B

$$L = I\omega = (20.5)(1.75) = 35.9 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$L = 35.9 \text{ kg}\cdot\text{m}^2/\text{s}$$

(d) Find the magnitude of the torque exerted (about point P) on the ball at point C



$$\begin{aligned} |\vec{\tau}| &= |\vec{r} \times \vec{F}| = rF \sin\theta \\ &= Lmg \sin\theta \\ &= (3.2)(2)(9.8) \sin 60^\circ \\ &= 54.3 \text{ N}\cdot\text{m} \end{aligned}$$

$$\tau = 54.3 \text{ N}\cdot\text{m}$$

**QUESTION 6 (20 %)**

A block of mass  $m = 680$  g is fastened to a spring whose spring constant is  $k = 65$  N/m. The block is pulled a distance  $x = 11$  cm from its equilibrium position and then released from rest.

(a) What force does the spring exert on the block just before it is released?

$$F = |-kx| = |-(65)(0.11)| = 7.15 \text{ N}$$

$$F = 7.15 \text{ N}$$

(b) What is the amplitude of the oscillation?

Since maximum distance is given as  $x = 11$  cm

$$A = x = 11 \text{ cm} = 0.11 \text{ m}$$

$$A = 0.11 \text{ m}$$

(c) What is the period of the oscillation?

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.68}} = 9.78 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{9.78} = 0.64 \text{ s}$$

$$T = 0.64 \text{ s}$$

(d) What is maximum speed of the oscillating block?

$$v_{\max} = \omega A = (9.78)(0.11) = 1.08 \text{ m/s}$$

$$v_{\max} = 1.08 \text{ m/s}$$