UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
Second Midterm Exam *FALL SEMESTER*
Date: 25/12/2007 Time: 100 min.

Name	Surname	Dep.	Signature
—	SOLUTIONS		—

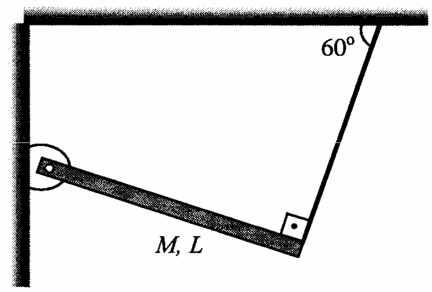
Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
OUT OF	100

*** **SOLVE ONLY 5 OUT OF 6 QUESTIONS** ***

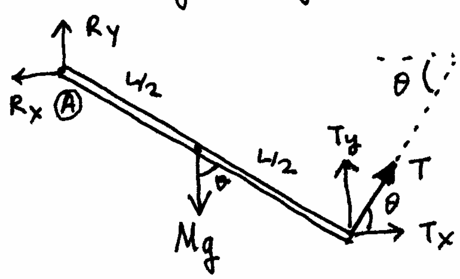
- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $g = 9.8\ \text{m/s}^2$

QUESTION 1 (20 %)

A uniform rod of mass $M = 2.0$ kg and length L is attached to a wall by a hinge and connected to a ceiling by a string. The rod is in equilibrium as shown in the figure. Find the tension in the string and the components of the reaction force exerted by the hinge on the rod.



Free body diagram of the rod:



Equilibrium conditions:

$$\sum F_x = T_x - R_x = 0 \qquad (1)$$

$$\sum F_y = R_y + T_y - Mg = 0 \qquad (2)$$

$$\sum \tau_{A} = TL - Mg\sin\theta \frac{L}{2} = 0 \qquad (3)$$

From eqn (3) : $T = \dfrac{Mg\sin\theta}{2} = \dfrac{(2.0)(9.8)\sin 60°}{2} = 8.5\ N$

$T_x = T\cos\theta = (8.5)\cos 60° = 4.3\ N$

$T_y = T\sin\theta = (8.5)\sin 60° = 7.4\ N$

From eqn (2) : $R_y = Mg - T_y = (2)(9.8) - 7.4 = 12.2\ N$

From eqn (1) : $R_x = T_x = 4.3\ N$

Tension:

$\boxed{T = 8.5\ N}$

Componets of the reaction Force:

$\boxed{R_x = 4.3\ N}$

$\boxed{R_y = 12.2\ N}$

QUESTION 2 (20 %)

Two balls made from clay of masses $m_1 = 1.0$ kg and $m_2 = 2.5$ kg travel with speeds of $v_1 = 2.0$ m/s and $v_2 = 1.0$ m/s and collide at the origin as shown in Fig I. After the collision, they stick together and move in the direction shown in Fig II.

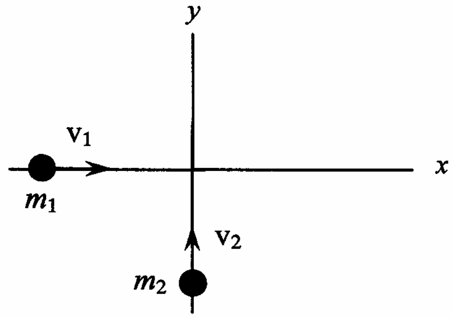


Fig I. *Before collision*

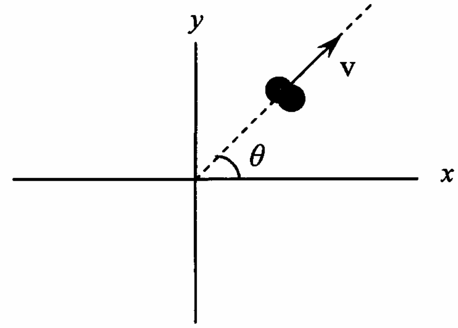


Fig II. *After collision*

(a) Derive all the necessary equations to find the direction and magnitude of the common velocity after collision

Conservation of linear momentum:

$$\vec{P}_i = \vec{P}_f$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2)\vec{v}$$

in x-direction: $m_1 v_1 = (m_1 + m_2) v \cos\theta$ (1)

in y-direction: $m_2 v_2 = (m_1 + m_2) v \sin\theta$ (2)

(b) Calculate the direction (θ) and magnitude of the common velocity (v) after the collision

Dividing eqn (2) by (1) gives:

$$\frac{v \sin\theta}{v \cos\theta} = \tan\theta = \frac{m_2 v_2}{m_1 v_1} = \frac{(2.5)(1.0)}{(1.0)(2.0)} = 1.25$$

$$\therefore \theta = \tan^{-1}(1.25) = 51.3^\circ$$

using eqn (1)

$$v = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta} = \frac{(1.0)(2.0)}{(1.0 + 2.5)\cos 51.3^\circ} = 0.9 \text{ m/s}$$

Direction:

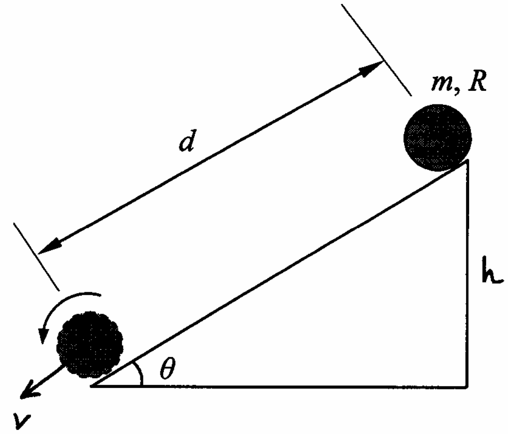
$\boxed{\theta = 51.3^\circ}$

Common velocity:

$\boxed{v = 0.9 \text{ m/s}}$

QUESTION 3 (20 %)

A ping-pong ball of mass $m = 3.0$ g and radius $R = 2.5$ cm is released from the top of an inclined plane of inclination angle $\theta = 30.0^{\circ}$ at $t = 0$ as shown in the figure. It rolls down without slipping and reaches the bottom of the plane at $t = 2.0$ s after traveling a distance $d = 5.9$ m. Find the rotational inertia (about its center of mass) of the ball.



The distance d, time t and acceleration of center of mass are related as follows:

$$d = \frac{1}{2} a_{cm} t^2 \quad \longrightarrow \quad a_{cm} = \frac{2d}{t^2}$$

The velocity of the ball at the end of the track is:

$$v^2 = \cancel{v_0^2}^{\,0} + 2 a_{cm} d = \frac{4d^2}{t^2} \qquad (*)$$

The conservation of mechanical energy:

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$mgd\sin\theta = \frac{1}{2} m v^2 + \frac{1}{2} I \frac{v^2}{R^2}$$

$$= \frac{v^2}{2}\left(m + \frac{I}{R^2}\right)$$

$$= \frac{4d^2}{2t^2}\left(m + \frac{I}{R^2}\right)$$

Solving for I

$$I = mR^2\left(\frac{g\sin\theta\, t^2}{2d} - 1\right)$$

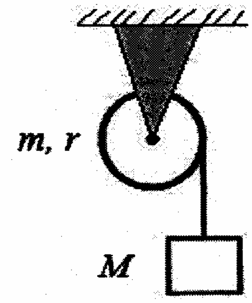
$$= (3.0\times10^{-3})(2.5\times10^{-2})^2\left[\frac{9.8\times\sin 30\times 2^2}{2\times 5.9} - 1\right]$$

$$= 1.2\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$\boxed{I = 1.2\times10^{-6}\ \text{kg}\cdot\text{m}^2}$$

QUESTION 4 (20 %)

A solid cylindrical drum of m = 10.0 kg and radius r = 0.5 m is rotating about its cylindrical axis under the influence of a force produced by a block of mass M = 3.0 kg attached to a cord wound around the drum as shown in the figure.

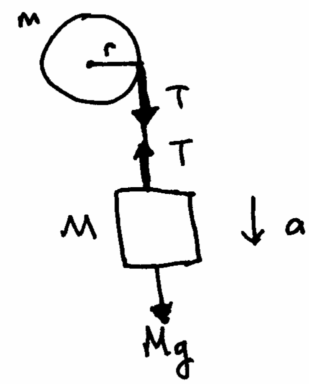


(The moment of inertia for a solid drum rotating about its axis is given as $I = mr^2/2$)

(a) Find the acceleration of the block

Newton's 2nd law for the block:

$$Mg - T = Ma \qquad (1)$$



Torque about axis of the drum:

$$\tau = I\alpha$$

$$Tr = \frac{mr^2}{2}\frac{a}{r}$$

$$T = \frac{ma}{2} \qquad (2)$$

Using (1) and (2) $\quad a = \dfrac{Mg}{M + m/2} = \dfrac{3.0 \times 9.8}{3.0 + 10.0/2} = 3.7$ m/s

$$\boxed{a = 3.7\ m/s^2}$$

(b) Find the angular acceleration of the drum

$$\alpha = \frac{a}{r} = \frac{3.7}{0.5} = 7.4\ rad/s^2$$

$$\boxed{\alpha = 7.4\ rad/s^2}$$

(c) Find the torque exerted on the on the drum

$$\tau = Tr = \left(\frac{ma}{2}\right) r = \left(\frac{10.0 \times 3.7}{2}\right) 0.5$$

$$= 9.3\ N \cdot m$$

$$\boxed{\tau = 9.3\ N \cdot m}$$

QUESTION 5 (20 %)

A system consists of two particles of masses $m_1 = 2.0$ kg and $m_2 = 4.0$ kg. Their position vectors are given as $\mathbf{r}_1 = 3t\mathbf{i} - 9t^2\,\mathbf{j}$ (m) and $\mathbf{r}_2 = (3t-2)\mathbf{i} + 6t\mathbf{j}$ (m), respectively, where time t is in seconds. Answer the followings in vector form.

(a) Find the position of center of mass the system as a function of time

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{(2.0)\left[3t\hat{i} - 9t^2\hat{j}\right] + (4.0)\left[(3t-2)\hat{i} + 6t\hat{j}\right]}{2.0+4.0}$$

$$= \frac{3t\hat{i} - 9t^2\hat{j}}{3.0} + \frac{2.0\left[(3t-2)\hat{i} + 6t\hat{j}\right]}{3.0}$$

$$= t\hat{i} - 3t^2\hat{j} + (2t - 4/3)\hat{i} + 4t\hat{j}$$

$$= (3t - 4/3)\hat{i} + (4t - 3t^2)\hat{j} \quad (m)$$

$$\vec{r}_{cm} = \boxed{(3t - 4/3)\hat{i} + (4t - 3t^2)\hat{j}}\,(m)$$

(b) Find the velocity of center of mass of the system as a function of time

$$\vec{V}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{d}{dt}\left[(3t - 4/3)\hat{i} + (4t - 3t^2)\hat{j}\right]$$

$$= 3\hat{i} + (4 - 6t)\hat{j} \quad (m/s)$$

$$\boxed{\vec{V}_{cm} = 3\hat{i} + (4-6t)\hat{j}}\,(m/s)$$

(c) Find the acceleration of center of mass of the system

$$\vec{a}_{cm} = \frac{d\vec{V}_{cm}}{dt} = \frac{d}{dt}\left[3\hat{i} + (4-6t)\hat{j}\right]$$

$$= -6\hat{j} \quad (m/s^2)$$

$$\boxed{\vec{a}_{cm} = -6\hat{j}\ (m/s^2)}$$

(d) Find the net external force acting on the system

$$\vec{F}_{net} = (m_1 + m_2)\,\vec{a}_{cm} = (2.0 + 4.0)(-6.0\hat{j}) = -36\hat{j}\ (N)$$

$$\boxed{\vec{F}_{net} = -36\hat{j}\ (N)}$$

QUESTION 6 (20 %)
An object of mass m = 0.4 kg oscillates with simple harmonic motion of amplitude of 3.0 cm along the x-axis under the action of the elastic force (force constant k = 10.0 N/m). At t = 0 the displacement of the particle is x_o = 1.5 cm. Assume that the general form of the displacement is $x = A\cos(\omega t + \varphi)$.

(a) Find the frequency of the oscillation

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10.0}{0.4}} = 5.0 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{5.0}{2\pi} = 0.8 \text{ Hz}$$

$$\boxed{f = 0.8 \text{ Hz}}$$

(b) Find the phase constant

$$x = A\cos(\omega t + \phi)$$

$$x = (3\text{ cm})\cos(5t + \phi)$$

$$\phi = \cos^{-1}(1/2) = 60^\circ = \pi/3 \text{ rad}$$

at $t = 0 \rightarrow x = x_o = 1.5$ cm

$$\therefore \quad 1.5 \text{ cm} = (3.0 \text{ cm})\cos(\phi)$$

$$\boxed{\phi = \pi/3 \text{ rad}}$$

(c) Find the speed of the object at t = 3 s

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[(3\text{ cm})\cos(5t + \pi/3)\right]$$

$$= -(15 \text{ cm/s})\sin(5t + \pi/3)$$

at $t = 3$s $\quad v(3) = -(15 \text{ cm/s})\sin(15 + \pi/3) = 5.0 \text{ cm/s}$

$$\boxed{v = 5.0 \text{ cm/s}}$$

(d) Find the maximum value of the speed and acceleration

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[(3\text{ cm})\cos(5t + \pi/3)\right] = (-15 \text{ cm/s})\sin(5t + \pi/3)$$

$$\Rightarrow \quad v_{max} = 15 \text{ cm/s}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[(-15 \text{ cm/s})\sin(5t + \pi/3)\right]$$

$$= (-75 \text{ cm/s}^2)\cos(5t + \pi/3)$$

$$\Rightarrow \quad a_{max} = 75 \text{ cm/s}^2$$

$$\boxed{v_{max} = 15 \text{ cm/s}}$$

$$\boxed{a_{max} = 75 \text{ cm/s}^2}$$