UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
First Midterm Exam FALL SEMESTER
Date: 08/11/2007 Time: 100 min.

Name	Surname	Dep.	Signature
— SOLUTIONS —			

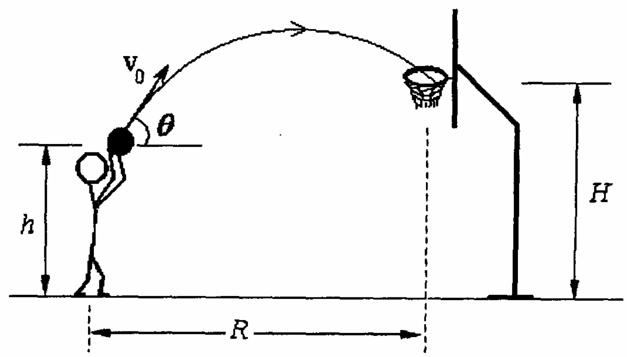
Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
OUT OF	100

*** SOLVE ONLY 5 OUT OF 6 QUESTIONS ***

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $g = 9.8\ m/s^2$; $\sin 40° = 0.643$ $\cos 40° = 0.766$

QUESTION 1 (20 %)

A basketball player who is $h = 2$ m tall is standing on the floor $R = 10$ m from the basket, as in Figure. If he shoots the ball at a $\theta = 40°$ angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is $H = 3.05$ m.



Let t_f be the time of flight for the ball.

- $x = v_0 \cos\theta\, t$

$$R = v_0 \cos\theta\, t_f \longrightarrow t_f = \frac{R}{v_0 \cos\theta}$$

- $y = y_0 + v_0 \sin\theta\, t - \frac{1}{2} g t^2$

$$H = h + v_0 \sin\theta\, t_f - \frac{1}{2} g t_f^2$$

Substituting t_f yields:

$$H = h + v_0 \sin\theta \left(\frac{R}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{R}{v_0 \cos\theta}\right)^2$$

Solving for v_0:

$$v_0 = \left[\frac{g R^2}{2 \cos^2\theta \,(h + R\tan\theta - H)}\right]^{1/2}$$

$$= \left[\frac{9.8 \times 10^2}{2 \cos^2 40° \,(2 + 10 \tan 40° - 3.05)}\right]^{1/2}$$

$$= 10.67\ m/s$$

$$\boxed{v_0 = 10.67\ m/s}$$

QUESTION 2 (20 %)

Given the displacement vectors $\vec{\mathbf{A}} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ (m) and $\vec{\mathbf{B}} = 2\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$ (m)

(a) Find a vector $\vec{\mathbf{C}}$ such that $2\vec{\mathbf{A}} - \vec{\mathbf{B}} - \vec{\mathbf{C}} = \mathbf{0}$

$$2\vec{A} - \vec{B} - \vec{c} = 0 \rightarrow \vec{c} = 2\vec{A} - \vec{B}$$
$$= 2(3\hat{i} - 4\hat{j} + 4\hat{k}) - (2\hat{i} + 3\hat{j} - 7\hat{k})$$
$$= (6-2)\hat{i} + (-8-3)\hat{j} + (8+7)\hat{k}$$
$$= 4\hat{i} - 11\hat{j} + 15\hat{k} \text{ (m)}$$

$$\boxed{\vec{c} = 4\hat{i} - 11\hat{j} + 15\hat{k} \text{ (m)}}$$

(b) Using the definition of dot product, find the angle in degrees between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

$$|\vec{A}| = A = \sqrt{3^2 + (-4)^2 + 4^2} = \sqrt{41} \text{ m}$$
$$|\vec{B}| = B = \sqrt{2^2 + 3^2 + (-7)^2} = \sqrt{62} \text{ m}$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{(3\hat{i} - 4\hat{j} + 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 7\hat{k})}{\sqrt{41}\,\sqrt{62}} = \frac{-34}{50.42}$$

$$\therefore \theta = \cos^{-1}(-34/50.42) = 132.4^\circ$$

$$\boxed{\theta = 132.4^\circ}$$

(c) Find the unit vector in the direction of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 4 \\ 2 & 3 & -7 \end{vmatrix} = 16\hat{i} + 29\hat{j} + 17\hat{k} \text{ (m}^2\text{)}$$

$$\hat{u} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{16\hat{i} + 29\hat{j} + 17\hat{k}}{\sqrt{16^2 + 29^2 + 17^2}} = 0.43\hat{i} + 0.78\hat{j} + 0.46\hat{k}$$

$$\boxed{\hat{\mathbf{u}} = 0.43\hat{i} + 0.78\hat{j} + 0.46\hat{k}}$$

QUESTION 3 (20 %)

Table gives data on the position (measured in meter) of a particle moving along a straight line at various times (measured in seconds).

Assume that the particle acceleration is constant.

Time, t(s)	Position, x(m)
0.0	1.00
0.5	4.25
1.0	6.50
1.5	7.75
2.0	8.00
2.5	7.25
3.0	5.50
3.5	2.75
4.0	-1.00

(a) Find an expression as for the particle position as a function of time.

General equation: $x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$ we need to find x_0, v_0, a.

at $t=0s \rightarrow x(0) = x_0 + 0 \cdot v_0 + \frac{1}{2} a \cdot 0^2 = 1.00 \rightarrow x_0 = 1.00$ m

at $t=1s \rightarrow x(1) = 1 + v_0 + \frac{1}{2} a \rightarrow 1 + v_0 + a/2 = 6.50$ (1)

at $t=2s \rightarrow x(2) = 1 + 2v_0 + 2a \rightarrow 1 + 2v_0 + 2a = 8.00$ (2)

Solving v_0 and a from equation (1) and (2)

$v_0 = 7.5$ m/s

$a = -4.0$ m/s^2 $\therefore x(t) = 1 + 7.5t - 2t^2$ (meter)

$x(t) = 1 + 7.5t - 2t^2$ (m)

(b) What is the position of the particle at $t = 2.3$ s?

$$x(2.3) = 1 + 7.5(2.3) - 2(2.3)^2$$
$$= 7.67 \text{ m}$$

$x = 7.67$ m

(c) What is the average velocity of the particle between $t = 3.5$ s and $t = 1.0$ s?

$$v_{avr} = \frac{x(3.5) - x(1.0)}{3.5 - 1.0} = \frac{2.75 - 6.50}{2.5} = -1.50 \text{ m/s}$$

$v_{avr} = -1.50$ m/s

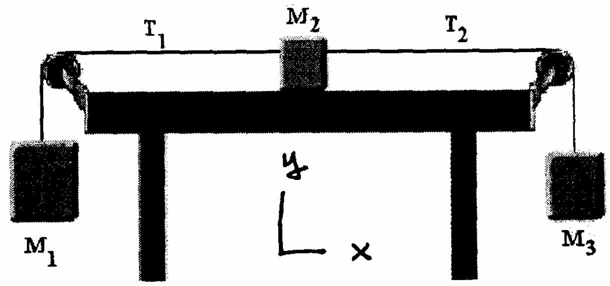
(d) What is the instantaneous velocity of the particle at $t = 2.0$ s?

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[1 + 7.5t - 2t^2\right]$$
$$= 7.5 - 4t$$
$$v(2) = 7.5 - 4(2)$$
$$= -0.50 \text{ m/s}$$

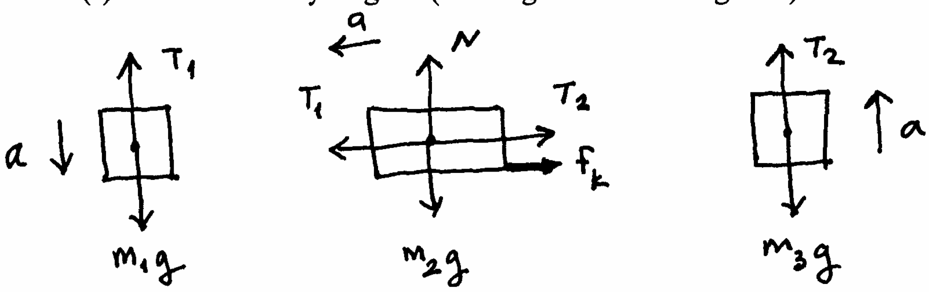
$v = -0.50$ m/s

QUESTION 4 (20 %)

Three blocks are connected on the table, as shown in Figure. The table surface is rough and has a coefficient of kinetic friction of 0.350. The blocks have masses of $M_1 = 4$ kg, $M_2 = 1$ kg, and $M_3 = 2$ kg, and the pulleys are frictionless.



(a) Draw a free-body diagram (showing all forces acting on it) for each of the objects.



$$f_k = \mu_k N$$
$$= \mu_k m_2 g$$

(b) Determine the acceleration of the system and its direction.

Newton's 2^{nd} law for each block is as follows:

$$m_1 g - T_1 = m_1 a \qquad (1)$$

$$T_1 - T_2 - f_k = m_2 a \qquad (2)$$

$$T_2 - m_3 g = m_3 a \qquad (3)$$

Adding (1) + (2) + (3)

$$m_1 g - \mu_k m_2 g - m_3 g = (m_1 + m_2 + m_3) a$$

$$\therefore \quad a = \left(\frac{m_1 - \mu_k m_2 - m_3}{m_1 + m_2 + m_3}\right) g$$

$$= \left(\frac{4 - 0.35(1) - 2}{4 + 1 + 2}\right) 9.8$$

$$= 2.31 \text{ m/s}^2$$

in the direction of $-x$

$\boxed{a = 2.31 \text{ m/s}^2}$

(c) Determine the tensions in the two cords

from equation (1): $T_1 = m_1 (g - a) = 4(9.8 - 2.31) = 30$ N

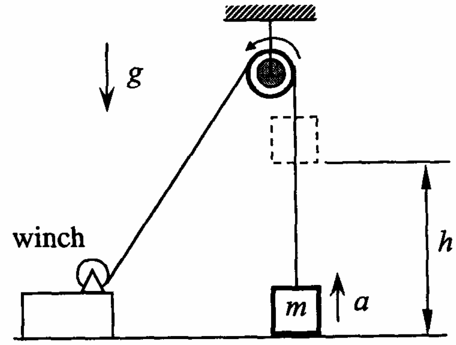
from equation (3): $T_2 = m_3 (g + a) = 2(9.8 + 2.31) = 24.22$ N

$\boxed{T_1 = 30.00 \text{ N}}$

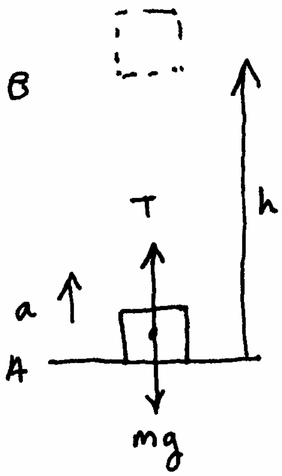
$\boxed{T_2 = 24.22 \text{ N}}$

QUESTION 5 (20 %)

A block of mass $m = 50$ kg is pulled up at a constant acceleration of $a = 2$ m/s^2 by means of a small winch as shown in Figure. The block being initially at rest rises to a height of $h = 16$ m. Ignore the mass of the pulley and all possible frictions in the system.



(a) Find the work done by the winch at the end of the rise



Newton's 2nd law:

$$T - mg = ma$$

$$T = m(a+g) = 50(2+9.8) = 590\ N$$

$$W_w = T h \cos 0^\circ = (590)(16)(1) = 9440\ J$$

$\boxed{W_w = 9440\ J}$

(b) Find the work done by the gravity (i.e. weight of the block) at the end of the rise

$$W_g = mgh\cos 180^\circ = -mgh = -(50)(9.8)(16) = -7840\ J$$

$\boxed{W_g = -7840\ J}$

(c) Find the average power delivered by the winch during the period of the total rise

Time required to come point B is:

$$h = \cancel{v_0 t}^{\,0} + \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{(2)(16)}{2}} = 4\ s$$

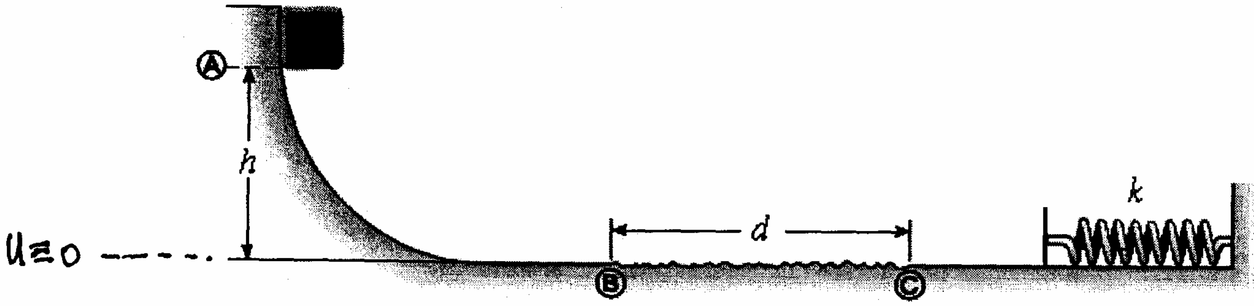
Average power:

$$\bar{P} = \frac{W_w}{t} = \frac{9440\ J}{4\ s} = 2360\ W$$

$\boxed{P_{AVR} = 2360\ W}$

QUESTION 6 (20 %)

A m = 10-kg block is released from point A whose height is h = 3 m, as in Figure given below. The track is frictionless except for the portion between B and C, which has a length of d = 2 m. The block travels down the track, hits a spring of force constant k = 2250 N/m, and compresses the spring x = 0.4 m from its equilibrium position before coming to rest momentarily. Then the block goes back and gains a new height between A and B, after passing through the points C and B respectively.



(a) Determine the coefficient of kinetic friction between the block and rough surface

work done by the friction is change in mechanical energy.

$$W_f = \Delta E$$

$$-f_k d = (U_f + \cancel{K_f}^{0}) - (U_i + \cancel{K_i}^{0})$$

$$-\mu_k m g d = \frac{1}{2} k x^2 - m g h$$

Solving for μ_k:

$$\mu_k = \frac{mgh - 1/2\, kx^2}{mgd}$$

$$= \frac{(10)(9.8)(3) - 1/2(2250)(0.4)^2}{(10)(9.8)(2)}$$

$$= 0.58$$

μ_k = 0.58

(b) Find the height to which the block rebounds back after passing through the points C and B.

$$W_f = \Delta E$$

$$-\mu_k m g d = m g H - \frac{1}{2} k x^2$$

$$\therefore \quad H = \frac{1/2\, kx^2 - \mu_k m g d}{mg}$$

$$= \frac{1/2\,(2250)(0.4)^2 - (0.58)(10)(9.8)(2)}{(10)(9.8)}$$

$$= 0.67 \text{ m}$$

$$= 67 \text{ cm}$$

H = 0.67 m