UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
Final Exam SUMMER SCHOOL

Date: 14/08/2006 Time: 110 min.

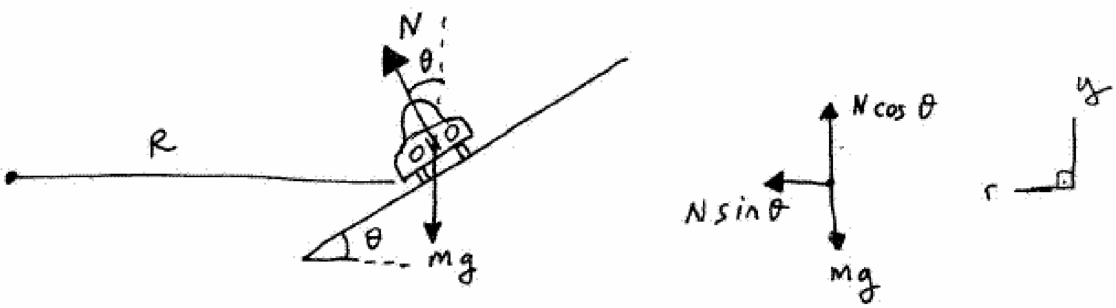
Name	Surname	Dep.	Signature
	Solutions !		

Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
Out of	120

- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Useful constants: $g = 9.8\ m/s^2$; $\sin30^\circ=0.5$ $\cos30^\circ=0.866$

QUESTION 1 (20)

An automobile makes a turn whose radius 28 m. The road is banked at an angle of 10°. We will assume that the friction between the tires and the road is zero. At what speed should the driver take the curve to avoid sliding off the road?



Only the component $N \sin\theta$ causes the centripedal acceleration. Hence Newton's 2nd law for radial direction:

(1) $\sum F_r = N \sin\theta = m v^2/R$

The car is in equilibrium in vertical direction

(2) $\sum F_y = N \cos\theta - mg = 0$

solving v from (1) and (2)

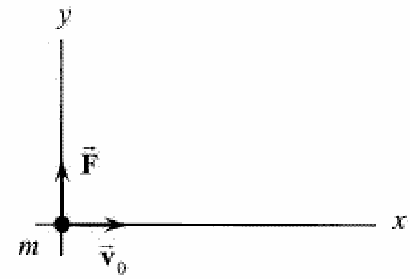
$$v = \sqrt{Rg \tan\theta} = \sqrt{(28)(9.8)\tan 10^\circ}$$

$$= 7\ m/s$$

$\boxed{v = 7\ m/s}$

QUESTION 2 (20)

A particle of mass $m = 2$ kg moves on xy plane under the influence of a constant force $\vec{\mathbf{F}} = 12\mathbf{j}$ (N). The particle is initially at the origin and its initial velocity vector at $t = 0$ is given by $\vec{\mathbf{v}}_0 = 3\mathbf{i}$ (m/s) as shown in Figure. Ignore the gravitational effects and answer the followings <u>in unit vector notation</u>.



(a) Find an expression for the velocity vector, $\vec{\mathbf{v}}$, of the ball as a function of time, t.

Acceleration of the particle:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{12\hat{j}}{2} = 6\hat{j} \ \text{m/s}^2$$

$$\vec{v} = \vec{v}_0 + \vec{a}\,t = 3\hat{i} + 6t\hat{j} \ \text{(m/s)}$$

$\boxed{\vec{\mathbf{v}} = 3\hat{i} + 6t\hat{j} \ \text{(m/s)}}$

(b) Find an expression for the position vector, $\vec{\mathbf{r}}$, of the particle as a function of time, t.

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$
$$= 0 + 3t\hat{i} + \frac{1}{2}6t^2\hat{j}$$
$$= 3t\hat{i} + 3t^2\hat{j} \ \text{(m)}$$

$\boxed{\vec{\mathbf{r}} = 3t\hat{i} + 3t^2\hat{j} \ \text{(m)}}$

(c) Find the angle in degrees between the position vector ($\vec{\mathbf{r}}$) and the velocity vector ($\vec{\mathbf{v}}$) at $t = 2$ s

$$\vec{r}(2) = 3(2)\hat{i} + 3(2)^2\hat{j} = 6\hat{i} + 12\hat{j} \ \text{(m)}$$
$$r = \sqrt{6^2 + 12^2} = \sqrt{180}$$

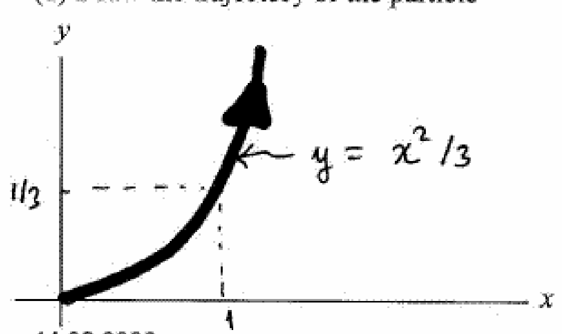
$$\vec{v}(2) = 3\hat{i} + 6(2)\hat{j} = 3\hat{i} + 12\hat{j} \ \text{(m/s)}$$
$$v = \sqrt{3^2 + 12^2} = \sqrt{153}$$

$$\cos\theta = \frac{\vec{r}\cdot\vec{v}}{rv} = \frac{(6\hat{i} + 12\hat{j})\cdot(3\hat{i} + 12\hat{j})}{\sqrt{180}\,\sqrt{153}} = \frac{162}{166} \approx 0.976$$

$$\therefore \theta = \cos^{-1}(0.976) = 12.6^\circ$$

$\boxed{\theta = 12.6^\circ}$

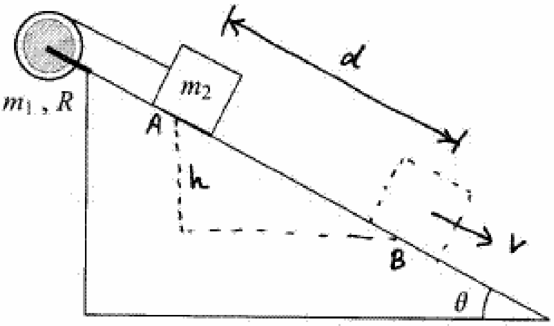
(d) Draw the trajectory of the particle



$$\vec{r} = 3t\hat{i} + 3t^2\hat{j}$$
$$x = 3t \rightarrow t = x/3$$
$$y = 3t^2 \rightarrow y = 3x^2/9$$
$$\therefore \boxed{y = \frac{x^2}{3}} \text{ a parabola}$$

QUESTION 3 (20)

A light cord is wrapped around a pulley of radius R = 10 cm and mass m_1 = 1 kg. The free end of the cord is attached to a block of mass m_2 = 2 kg. The block starts from rest and slides down on incline that makes an angle of θ = 37° with the horizontal as shown. The coefficient of kinetic friction between the block and incline is μ = 0.25.



(Moment of inertia of the pulley about its CM is $I = m_1R^2/2$)

(a) Calculate block's speed after it has traveled a distance d = 2.5 m down the incline.

change in mechanical energy:

$$\Delta E = -fd = (K_B + \cancel{U_B}^{0}) - (\cancel{K_A}^{0} + U_A)$$

$$-\mu m_2 g \cos\theta d = \frac{1}{2} m_2 V^2 + \frac{1}{2} I \omega^2 - m_2 g h$$

$$= \frac{1}{2} m_2 V^2 + \frac{1}{2}\left(\frac{m_1R^2}{2}\right)\frac{V^2}{R^2} - m_2 g d \sin\theta$$

$\therefore V = 4$ m/s

Solving for V:

$$v = \frac{m_2 g d(\sin\theta - \mu\cos\theta)}{m_2/2 + m_1/4} = \frac{2\times 9.8\times 2.5(\sin 37° - 0.25\cos 37°)}{2/2 + 1/4}$$

$\boxed{v = 4 \text{ m/s}}$

(b) Calculate the magnitude of the acceleration of the block.

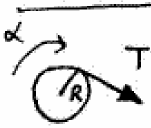
$$V_B^2 = \cancel{V_A^2}^{0} + 2ad$$

$$a = \frac{V_B^2}{2d} = \frac{4^2}{2(2.5)} = 3.2 \text{ m/s}^2$$

$\boxed{a = 3.2 \text{ m/s}^2}$

(c) Calculate the tension in the cord.

Method I for the pulley:

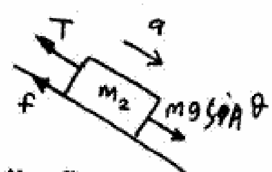


$$TR = I\alpha = \left(\frac{m_1R^2}{2}\right)\left(\frac{a}{R}\right)$$

$$T = \frac{m_1 a}{2} = \frac{(1)(3.2)}{2}$$

$$= 1.6 \text{ N}$$

Method II for the block:



$$m_2 g \sin\theta - T - f = m_2 a$$

$$T = m_2 g \sin\theta - \mu m_2 g \cos\theta - m_2 a$$

$$= m_2(g\sin\theta - \mu g\cos\theta - a)$$

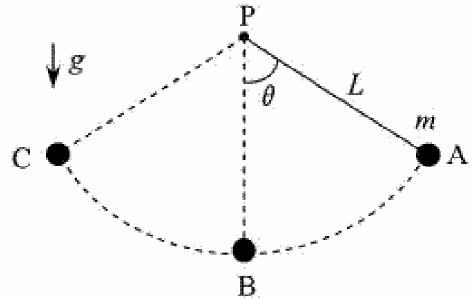
$$= 2(9.8\sin 37° - 0.25\times 9.8\cos 37° - 3.2)$$

$$= 1.6 \text{ N}$$

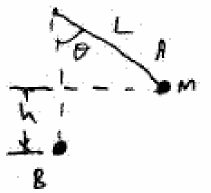
$\boxed{T = 1.6 \text{ N}}$

QUESTION 4 (20)

A pendulum consists of a bob of mass $m = 2$ kg attached to a light cord of length $L = 3.2$ m as shown in Figure. The bob is releases from rest (point A) when the cord makes an angle $\theta = 60^{\circ}$ with the vertical. The bob passes through its lowest point B and stops at point C. Ignore all possible frictions in the system.



(a) What is the speed of the bob at point B?



$$E_A = E_B$$

$$mgh = \frac{1}{2} m v_B^2$$

$$v_B = \sqrt{2gh} = \sqrt{2gL(1-\cos\theta)}$$

$$v_B = \sqrt{2 \times 9.8 \times 3.2(1-\cos 60^{\circ})}$$

$$= 5.6 \text{ m/s}$$

$\boxed{v_B = 5.6 \text{ m/s}}$

(b) What is the angular speed of the bob at point B?

$$\omega_B = \frac{v_B}{L} = \frac{5.6}{3.2} = 1.75 \text{ rad/s}$$

$\boxed{\omega_B = 1.75 \text{ rad/s}}$

(c) What is the magnitude of the linear momentum of the bob at point B?

$$P_B = m v_B = (2)(5.6) = 11.2 \text{ kg·m/s}$$

$\boxed{p_B = 11.2 \text{ kg·m/s}}$

(d) What is the magnitude of the angular momentum of the bob about the pivot (point P) at point B?

$$L_B = I_P \omega_B = (mL^2)\,\omega_B = [2 \cdot (3.2)^2](1.75) = 35.84 \text{ kg·m}^2\text{/s}$$

OR

$$L_B = P_B L = (11.2)(3.2) = 35.84 \text{ kg·m}^2\text{/s}$$

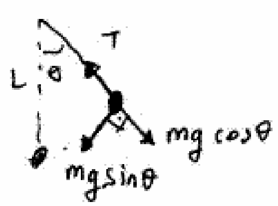
$\boxed{L_B = 35.8 \text{ kg·m}^2\text{/s}}$

(e) What is the magnitude of the radial acceleration of the bob at point B?

$$a_r = \frac{v_B^2}{L} = \frac{(5.6)^2}{3.2} = 9.8 \text{ m/s}^2$$

$\boxed{a_r = 9.8 \text{ m/s}^2}$

(f) What is the magnitude of the tangential acceleration of the bob at point B?



$$a_t = \frac{F_t}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$

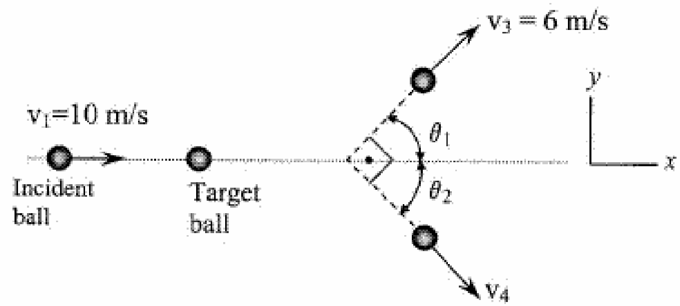
at point B $\theta = 0 \rightarrow \sin\theta = 0$

$$\therefore a_t = 0$$

$\boxed{a_t = 0}$

QUESTION 5 (20)

A ball with a mass *m* moving at a speed of $v_1 = 10$ m/s and strikes an identical stationary ball. After the collision, the angle between the balls is 90^0 and one of the ball has a speed of $v_3 = 6$ m/s as shown in below figure. Assume that the collision is completely elastic.



Before the collision *After the collision*

(a) What is the velocity of the other ball?

Kinetic energy is conserved since the collision is elastic.

$$K_1 + \cancel{K_2}^{\,0} = K_3 + K_4$$

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_3^2 + \frac{1}{2} m v_4^2$$

$$v_1^2 = v_3^2 + v_4^2$$

$$v_4 = \sqrt{v_1^2 - v_3^2} = \sqrt{10^2 - 6^2} = 8 \text{ m/s}$$

$\boxed{v_4 = 8 \text{ m/s}}$

(b) Find the angles (θ_1 and θ_2) of each ball with x-axis? (Hint: $\sin\alpha = \cos\beta$ if $\alpha + \beta = 90^0$)

Method I Conservation of momentum

x dir: $m v_1 = m v_3 \cos\theta_1 + m v_4 \cos\theta_2$ (1)

y dir: $0 = m v_3 \sin\theta_1 - m v_4 \sin\theta_2$ (2)

$v_1 = v_3 \cos\theta_1 + v_4 \cos\theta_2$ (3)

$0 = v_3 \cos\theta_2 - v_4 \cos\theta_1$ (4)

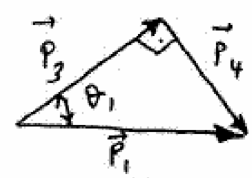
Solving for $\cos\theta_1$:

$$\cos\theta_1 = \frac{v_1 v_3}{v_3^2 + v_4^2} = \frac{10 \times 6}{6^2 + 8^2} = 0.6$$

$$\theta_1 = \cos^{-1}(0.6) = 53^\circ$$

$$\theta_2 = 90^\circ - 53^\circ = 37^\circ$$

Method II A right angle triangle



$$\cos\theta_1 = \frac{|\vec{P}_3|}{|\vec{P}_1|} = \frac{m v_1}{m v_3} = \frac{v_1}{v_3} = \frac{6}{10} = 0.6$$

$$\theta_1 = \cos^{-1}(0.6) = 53^\circ$$

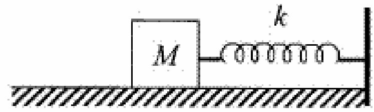
$$\theta_2 = 37^\circ$$

$\boxed{\theta_1 = 53^\circ}$

$\boxed{\theta_2 = 37^\circ}$

QUESTION 6 (20)

A 2-kg cube attached to a spring oscillates about $x = 0$ on a smooth surface as shown in Figure. Its position is given by:



$$x(t) = (5\text{ cm})\cos(2t + \pi)$$

where t is measured in seconds.

(a) What is the frequency of the cube?

$$X = A\cos(\omega t + \phi) \quad (1)$$
$$X = 5\cos(2t + \pi) \quad (2)$$

comparing (1) and (2)

$A = 5\text{ cm}$; $\omega = 2\text{ rad/s}$; $\phi = \pi$

$$f = \frac{\omega}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} \approx 0.32\ s$$

$\boxed{f = 0.32\ s}$

(b) What is the force constant of the spring?

$$\omega^2 = \frac{k}{m} \rightarrow k = m\omega^2 = (2)(2)^2 = 8\ N/m$$

$\boxed{k = 8\ N/m}$

(c) What is the velocity and acceleration of the cube at $t = 3$ s?

$$v = \frac{dx}{dt} = \frac{d}{dt}\left[5\cos(2t+\pi)\right] = -\left(10\ \frac{cm}{s}\right)\sin(2t+\pi)$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[-10\sin(2t+\pi)\right] = -\left(20\ \frac{cm}{s^2}\right)\cos(2t+\pi)$$

at $t = 3$ s

$$v(3) = -10\sin(2\cdot 3+\pi) = -2.8\ cm/s$$
$$a(3) = -20\cos(2\cdot 3+\pi) = +19.2\ cm/s^2$$

$\boxed{v = -2.8\ cm/s}$

$\boxed{a = +19.2\ cm/s^2}$

(d) What is the kinetic energy in Joules of the cube when it is at $x = 3$ cm?

velocity of a simple harmonic oscillator is:

$$v^2 = \frac{k}{m}(A^2 - x^2) \quad \text{(see lecture notes)}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{k}{m}(A^2 - x^2)$$
$$= \frac{1}{2}k(A^2 - x^2)$$
$$= \frac{1}{2}8(0.05^2 - 0.03^2)$$
$$= 6.4\times 10^{-3}\ J$$

$\boxed{K = 6.4\times 10^{-3}\ J}$