UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
Final Exam Questions FALL SEMESTER
Date: 17/01/2007 Time: 110 min.

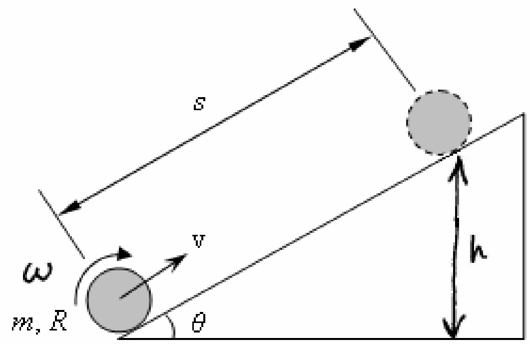
Name	Surname	Student No	Dep.	Signature
— SOLUTIONS —				

Ques.	Mark
1	
2	
3	
4	
5	
6	xxxxx
Total	
OUT OF	100

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $g = 9.8\ m/s^2$; $\sin 37^\circ = 0.6$, $\cos 37^\circ = 0.8$; $\sin 20^\circ = 0.34$, $\cos 20^\circ = 0.94$

QUESTION 1 (20 %)

A tennis ball of mass m and radius $R = 3$ cm rolls up without slipping an inclined plane of inclination angle $\theta = 37^\circ$. At the bottom of the incline the center of mass velocity of the ball is v = 10 m/s. The ball stops after traveling a distance s on the plane. The moment of inertia of the ball is given by $I = 2mR^2/3$.



(a) How far does the ball travel up the plane?

Mechanical energy is conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\frac{v^2}{R^2} = mgs\sin\theta$$

or

$$s = \frac{5v^2}{6g\sin\theta} = \frac{(5)(10^2)}{(6)(9.8)\sin 37} = 14.2\ m$$

$\boxed{s = 14.2\ m}$

(b) How many revolutions does the ball make until it stops?

$$\text{Number of revolutions} = \frac{s}{2\pi R} = \frac{14.2}{(2\pi)(0.03)} = 75\ rev$$

$\boxed{75\ rev}$

QUESTION 2 (20 %)

A particle of mass $m = 2$ kg moves in the xy plane. The position vector of the particle is given by:

$$\vec{\mathbf{r}} = (t-2)\,\mathbf{i} + (2t+1)\,\mathbf{j} \quad \text{(meter)}$$

where t is measured in seconds.

(a) Find the linear momentum vector of the particle

$$\vec{p} = m\vec{v} = m\frac{d\vec{r}}{dt} = (2)\frac{d}{dt}\left[(t-2)\hat{\imath} + (2t+1)\hat{\jmath}\right]$$

$$= 2\,(\hat{\imath} + 2\hat{\jmath})$$

$$= 2\hat{\imath} + 4\hat{\jmath} \;\; kg\cdot m/s$$

$$\boxed{\vec{\mathbf{P}} = 2\hat{\imath} + 4\hat{\jmath} \;\; kg\cdot\frac{m}{s}}$$

(b) Find the angular momentum vector of the particle about the origin

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= \left[(t-2)\hat{\imath} + (2t+1)\hat{\jmath}\right] \times \left[2\hat{\imath} + 4\hat{\jmath}\right]$$

$$= 4(t-2)\hat{k} - 2(2t+1)\hat{k}$$

$$= (4t - 8 - 4t - 2)\hat{k}$$

$$= -10\,\hat{k} \quad kg\cdot m^2/s$$

$$\boxed{\vec{\mathbf{L}} = -10\,\hat{k} \;\; kg\cdot\frac{m^2}{s}}$$

(c) Find the torque exerted on the particle about the origin

Method I:

$$\vec{\tau} = \frac{d\vec{L}}{dt} = 0$$

Method II:

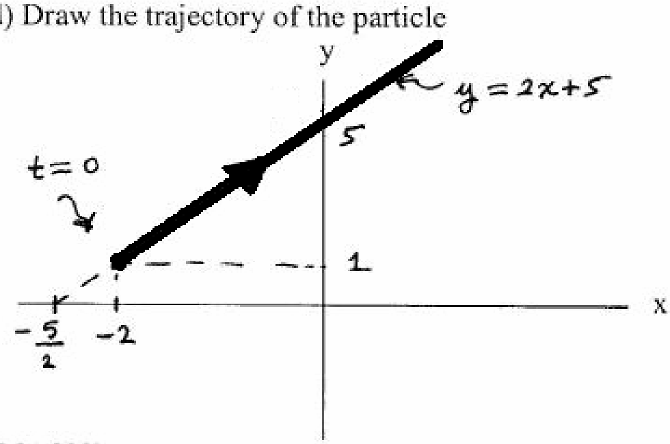
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$= \vec{r} \times \frac{d\vec{p}}{dt}$$

$$= 0 \qquad \text{since } \frac{d\vec{p}}{dt} = 0$$

$$\boxed{\vec{\tau} = \quad 0}$$

(d) Draw the trajectory of the particle



$$x = t-2 \;\rightarrow\; t = x+2$$

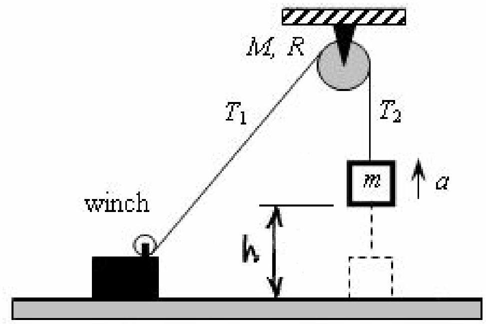
$$y = 2t+1$$

$$y = 2(x+2)+1$$

$$\boxed{y = 2x+5}$$

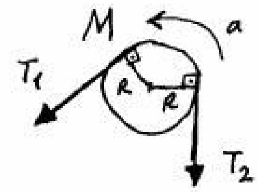
$$\text{at } t=0 \;\begin{cases} x = -2\text{ m} \\ y = 1\text{ m}\end{cases}$$

QUESTION 3 (20 %)



A block of mass m = 15 kg is elevated by a winch via a light cord that is wrapped around the rim of a pulley of mass M = 10 kg and radius R as shown in the Figure. The block is initially at rest and moves upward during 3 s with constant acceleration of a = 2.2 m/s^2.
(The rotational inertia of the pulley is $I = MR^2 / 2$)

(a) Determine the tensions T_1 and T_2 in the cord.



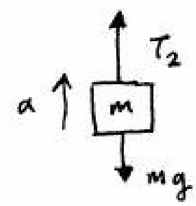
Newton's 2nd law

for the block : $T_2 - mg = ma \quad (1)$

for the pulley : $\tau = I\alpha$

$$(T_1 - T_2)R = \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right)$$

$$T_1 - T_2 = \frac{Ma}{2} \quad (2)$$



from eqn (1) : $T_2 = m(a+g) = 15(2.2+9.8) = 180\ N$

from eqn (2) : $T_1 = \frac{Ma}{2} + T_2 = \frac{(10)(2.2)}{2} + 180 = 191\ N$

T_1 = 191 N

T_2 = 180 N

(b) How much work is done by the winch on the system during the 3 s period?

work done by the winch = work done on the block + work done on pulley

$$W = T_2 h + \tau\theta$$
$$= T_2 h + (T_1 - T_2)R\frac{h}{R}$$
$$= T_2 h + T_1 h - T_2 h$$
$$= T_1 h$$
$$= T_1\left(\frac{1}{2}at^2\right)$$
$$= (191)\left(\frac{1}{2}\times 2.2\times 3^2\right)$$
$$= 1891\ J$$

W = 1891 J

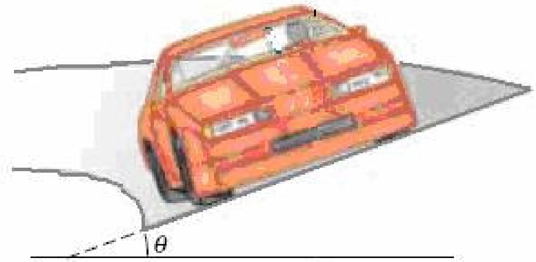
(c) What is the average power of the winch during the 3 s period?

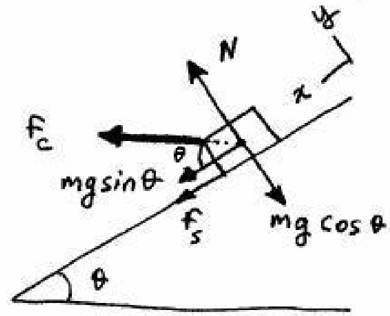
$$P_{AVR} = \frac{W}{\Delta t} = \frac{1891}{3} = 630.3\ Watt$$

P_{AVR} = 630.3 W

QUESTION 4 (20 %)

A civil engineer wishes to design a banked road with a curve for a highway in such a way that a car can move without sliding up or down on the road. Suppose the radius of the curve is 100 m and the curve is banked at the angle $\theta = 20^o$. If the designated speed for the road is 90 km/h to prevent the car sliding up, what must be the minimum value of the coefficient of static friction between the tires and road?





$V = 90$ km/h $= 25$ m/s

$f_c = \frac{mv^2}{R}$ is the centripedal force.

Newton's 2nd law:

$$\Sigma F_x = mg\sin\theta + f_s = f_c\cos\theta$$

$$mg\sin\theta + \mu_s N = \frac{mv^2}{R}\cos\theta \quad (1)$$

$$\Sigma F_y = N - mg\cos\theta = f_c\sin\theta$$

$$N - mg\cos\theta = \frac{mv^2}{R}\sin\theta \quad (2)$$

Solving N from eqn (2)

$$N = mg\cos\theta + \frac{mv^2}{R}\sin\theta \quad (3)$$

Subsituting (3) into (1) yields:

$$mg\sin\theta + \mu_s\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) = \frac{mv^2}{R}\cos\theta$$

$$\therefore \mu_s = \frac{\frac{v^2}{R}\cos\theta - g\sin\theta}{\frac{v^2}{R}\sin\theta + g\cos\theta}$$

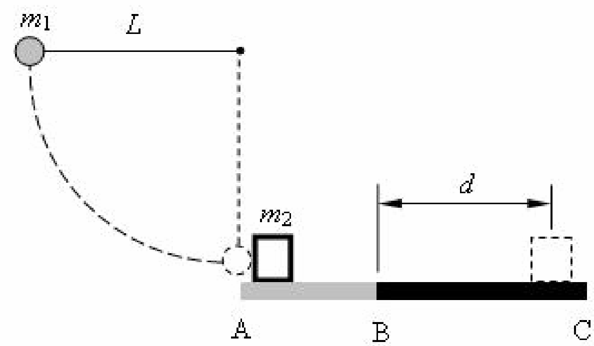
$$= \frac{25^2/100\ \cos 20 - 9.8\sin 20}{25^2/100\ \sin 20 + 9.8\cos 20}$$

$$= 0.22$$

$\mu_s = 0.22$

QUESTION 5 (20 %)

A steel ball of mass $m_1 = 0.5$ kg is fastened to a cord $L = 0.7$ m long and is released when the cord is horizontal. At the bottom of its path, the ball strikes a steel block of $m_2 = 2.5$ kg initially at rest on a smooth surface. The collision is completely elastic and the surface between A and B is frictionless. After the collision, the steel block travels the distance d on a rough surface and stops as seen in the figure. The coefficient of kinetic friction between the block and surface between B and C is $\mu_k = 0.15$.



(a) Find the speed of the steel ball just before the collision

conservation of mechanical energy:

$$m_1 g L = \frac{1}{2} m_1 v^2$$

$$v = \sqrt{2gL} = \sqrt{(2)(9.8)(0.7)} = 3.7 \text{ m/s}$$

v = 3.7 m/s

(b) Find the speeds of the steel ball and block just after the collision

conservation of momentum : $m_1 v = m_1 v_{1f} + m_2 v_{2f}$

conservation of kinetik en. : $\frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$

Hence, solving for the final velocities:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v = \left(\frac{0.5 - 2.5}{0.5 + 2.5}\right) 3.70 = -2.47 \text{ m/s}$$

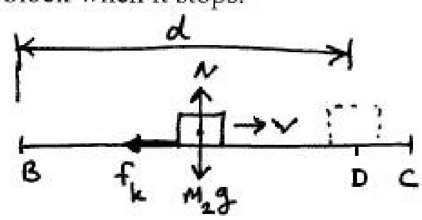
$$v_{2f} = \left(\frac{2 m_1}{m_1 + m_2}\right) v = \left(\frac{2 \times 0.5}{0.5 + 2.5}\right) 3.70 = +1.23 \text{ m/s}$$

v_{ball} = 2.47 m/s

v_{block} = 1.23 m/s

(c) Determine the distance traveled on the rough surface by the steel block when it stops.

work-energy theorem:



$$W = \Delta K$$

$$-f_k d = \frac{1}{2} \cancel{m_2 v_D^2}^{\,0} - \frac{1}{2} m_2 v_B^2$$

$$d = \frac{m_2 v_B^2}{2 f_k} = \frac{m_2 v_B^2}{2(\mu_k m_2 g)} = \frac{v_B^2}{2 \mu_k g} = \frac{(1.23)^2}{(2)(0.15)(9.8)} = 0.51 \text{ m}$$

d = 0.51 m