UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
Second Midterm Exam SUMMER SCHOOL

Date: 07/08/2006 Time: 100 min.

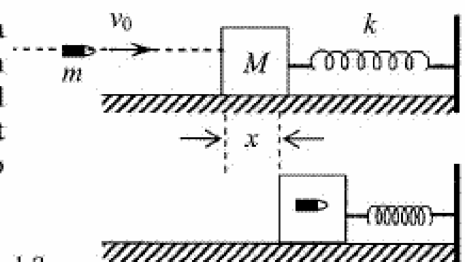
Name	Surname	Dep.	Signature
Solutions!			

Ques.	Mark
1	
2	
3	
4	
5	
6	
Total	
Out of	120

- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Useful constants: g = 9.8 m/s^2 ; sin12° = 0.21, cos12=0.98 cos53° = 0.6 sin53° = 0.8, cos60°=0.5 sin60°=0.866

QUESTION 1 (20)

A bullet of mass m moving with an initial speed v_0 is fired into a block of mass $M = \beta m$, where β is a positive constant, as shown in Figure. The block, initially at rest on a frictionless horizontal surface, is connected to a spring of force constant k. The bullet is stopped by the block and entire system moves a distance x to the right after impact.



(a) Find an expression for the speed of the bullet in terms of m, k, x and β.

Conservation of momentum:

$$m v_0 = (m + \beta m) V \rightarrow V = \frac{v_0}{1+\beta}$$

Conservation of energy after impact:

$$\frac{1}{2}(m + \beta m) V^2 = \frac{1}{2} k x^2 \qquad \therefore \quad v_0 = \sqrt{\frac{k}{m}(1+\beta)}\; x$$

$$\frac{1}{2} m (1+\beta) \frac{v_0^2}{(1+\beta)^2} = \frac{1}{2} k x^2$$

$$\boxed{v_0 = \left[(1+\beta)\, k/m\right]^{1/2} x}$$

(b) Calculate the fractional kinetic energy lost in the collision in terms of β?

Before impact: $K_i = \frac{1}{2} m v_0^2$

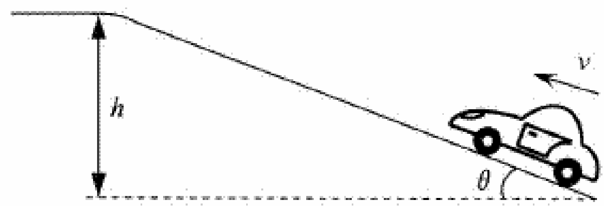
After impact: $K_f = \frac{1}{2} m (1+\beta) \frac{v_0^2}{(1+\beta)^2} = \frac{K_i}{1+\beta}$

$$f = \frac{K_f - K_i}{K_i} = \frac{K_i/(1+\beta) - K_i}{K_i} = \frac{-\beta}{1+\beta}$$

$$\boxed{f = -\frac{\beta}{1+\beta}}$$

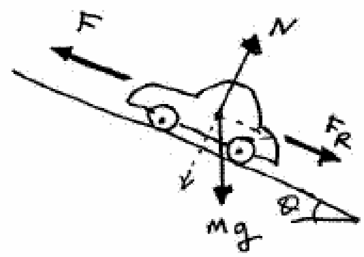
QUESTION 2 (20)

Consider a car of mass m = 1215 kg that is moving up a θ = 12° hill at a constant speed v = 36 km/h as shown in Figure. The total resistive force (road friction and air resistance) acting on the car for that speed is given by F_R = 1500 N and the height of the hill is h = 21 m.



(a) What power must be delivered by the car engine to travel the car at that constant speed?

Newton's 2nd law:



$$F - F_R - mg\sin\theta = 0$$

$$F = F_R + mg\sin\theta$$

$$= 1500 + (1215)(9.8)(\sin 12°)$$

$$= 4000\ N$$

v = 36 km/h = 10 m/s

Power: $P = Fv$

$$= (4000)(10)$$

$$= 40,000\ W$$

$$= 40\ kW \approx 54\ hp$$

P = 40 kW

(b) How much work is done by the force obtained from the car engine until the car reaches the top of the hill?

Total length of the way:

$$d = \frac{h}{\sin\theta} = \frac{21\ m}{\sin 12°} = \frac{21}{0.21} = 100\ m$$

$$W_F = F d \cos 0°$$

$$= (4000)(100)(1)$$

$$= 400,000\ J = 400\ kJ$$

W = 400 kJ

(c) How much work is done by the resistive force until the car reaches the top of the hill?

$$W_R = F_R\, d \cos 180°$$

$$= (1500)(100)(-1)$$

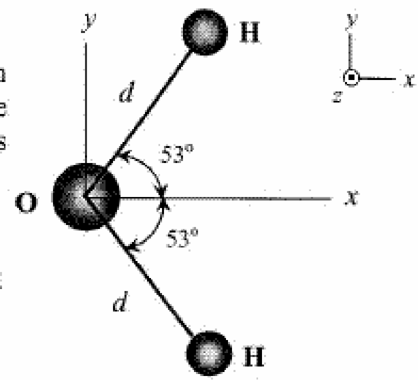
$$= -150,000\ J$$

$$= -150\ kJ$$

W = −150 kJ

QUESTION 3 (20)

A water molecule consists of an oxygen atom with two hydrogen atoms bound to it. The angle between the bonds is 106° and the bonds are $d = 0.1$ nm long. Assume that the atoms lie on xy plane as shown in Figure and the dimensions of the atoms are small.



1 nm = 10^{-9} m; $M_O = 16u$ and $M_H = 1u$
where u is the atomic mass unit and has the value $u = 1.66\text{x}10^{-27}$ kg

(a) Find the coordinates of the center of mass of the water molecule.

Total mass: $M = 16u + u + u = 18u = 3 \times 10^{-26}$ kg.

$$x_{cm} = \frac{m_o x_o + m_H x_H + m_H x_H}{M} = \frac{(16u)(0) + ud\cos 53° + ud\cos 53°}{18u} = \frac{d\cos 53°}{9} = \frac{0.1 \times 10^{-9} \times 0.6}{9} = 6.67 \times 10^{-12}\ \text{m}$$

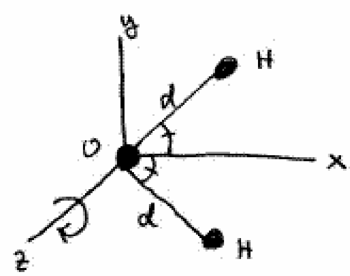
$$y_{cm} = \frac{m_o y_o + m_H y_H + m_H y_H}{M} = \frac{(16u)(0) + ud\sin 53° - ud\sin 53°}{18u} = 0$$

$$z_{cm} = \frac{m_o z_o + m_H z_H + m_H z_H}{M} = \frac{(16u)(0) + (u)(0) + (u)(0)}{18u} = 0$$

$x_{CM} = 6.67 \times 10^{-12}$ m	$y_{CM} = 0$	$z_{CM} = 0$

(b) What is the moment of inertia of about z-axis?

$$I_z = m_o r_o^2 + m_H r_H^2 + m_H r_H^2$$
$$= (16u)(0)^2 + ud^2 + ud^2$$
$$= 2ud^2$$
$$= (2)(1.66 \times 10^{-27})(0.1 \times 10^{-9})$$
$$= 3.32 \times 10^{-47}\ \text{kg}\cdot\text{m}^2$$

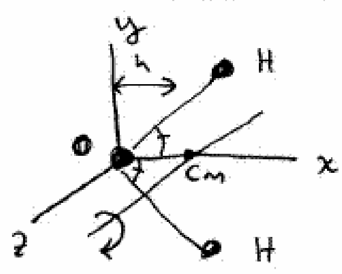


$I_z = 3.32 \times 10^{-47}$ kg·m²

(c) If the system rotates about the center of mass parallel to z axis, calculate the moment of inertia of the molecule about this axis.

Parallel axes theorem:

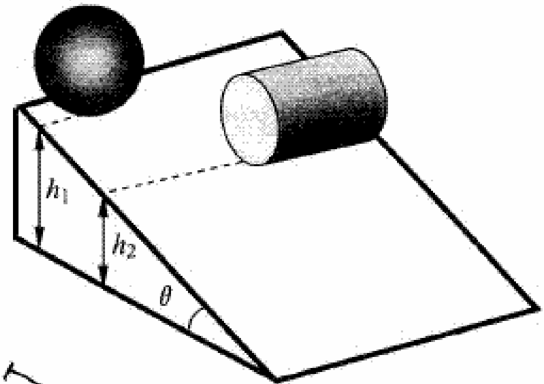
$$I_z = I_{cm} + Mh^2$$
$$I_{cm} = I_z - M x_{cm}^2$$
$$= 3.32 \times 10^{-47} - (3 \times 10^{-26})(6.67 \times 10^{-12})^2$$
$$= 3.19 \times 10^{-47}\ \text{kg}\cdot\text{m}^2$$



$h = x_{cm}$

$I_{CM} = 3.19 \times 10^{-47}$ kg·m²

QUESTION 4 (20)

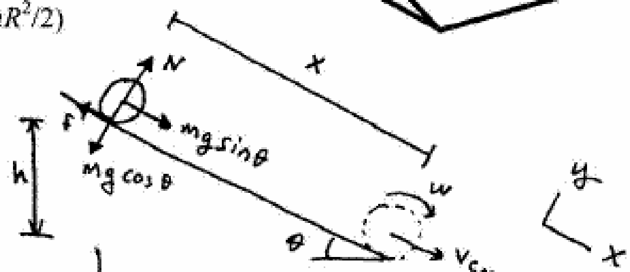


A solid sphere and a solid cylinder are placed on an incline as shown in Figure. They are released simultaneously from rest at the different heights (h_1 and h_2) and they roll without slipping.

If the initial height of the sphere is $h_1 = 1$ m, find the required initial height of the cylinder (h_2) such that the objects reach the bottom at the same time.

(About CM: $I_{\text{shpere}} = 2mR^2/5$ and $I_{\text{cylinder}} = mR^2/2$)

we need to find center of mass acceleration of the objects.



Method I: Newton's 2^{nd} law

$$\Sigma F_x = mg\sin\theta - f = m a_{cm} \quad (1)$$

$$\Sigma F_y = N - mg\cos\theta = 0 \quad (2)$$

$$\tau = I_{cm}\,\alpha$$

$$fR = I_{cm}\frac{a_{cm}}{R} \quad (3)$$

Use (1) and (3) to solve a_{cm}:

$$a_{cm} = \frac{g\sin\theta}{1 + \frac{I_{cm}}{mR^2}}$$

Method II: Conservation of energy

$$mgh = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm}\,w^2$$

$$= \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm}\frac{V_{cm}^2}{R^2}$$

solving for V_{cm}^2

$$V_{cm}^2 = \frac{2gh}{1 + \frac{I_{cm}}{mR^2}}$$

$$a_{cm} = \frac{V_{cm}^2 - V_{cm\,0}^2}{2x} = \frac{g\sin\theta}{1 + \frac{I_{cm}}{mR^2}}$$

for a sphere: $a_{cm} = \dfrac{g\sin\theta}{1 + \frac{2/5\,mR^2}{mR^2}} = \dfrac{5}{7}\,g\sin\theta$

for a cylinder: $a_{cm} = \dfrac{g\sin\theta}{1 + \frac{1/2\,mR^2}{mR^2}} = \dfrac{2}{3}\,g\sin\theta$

Time required to come to bottom can be found from:

$$x = \cancel{x_0}^{\,0} + \cancel{V_{cm\,0}}^{\,0}\,t + \frac{1}{2} a_{cm} t^2 \;\rightarrow\; t^2 = \frac{2x}{a_{cm}} = \frac{2h/\sin\theta}{a_{cm}}$$

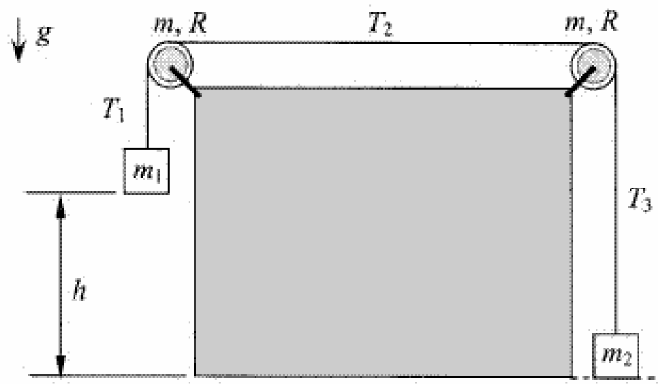
$$\therefore\quad t_1^2 = t_2^2$$

$$\frac{2h_1/\sin\theta}{\frac{5}{7}g\sin\theta} = \frac{2h_2/\sin\theta}{\frac{2}{3}g\sin\theta} \;\rightarrow\; h_2 = \frac{14}{15}h_1 \approx 0.93\,h_1 = 0.93\text{ m}$$

$$\boxed{h_2 = 0.93\text{ m}}$$

QUESTION 5 (20)

Two blocks having mass $m_1 = 18$ kg and $m_2 = 17$ kg are connected to each other by a light cord that passes over two identical, frictionless pulleys, each having mass $m = 5$ kg and radius R as shown in the Figure. The tensions in the cord are T_1, T_2 and T_3 respectively. The system is released at $t = 0$ s and the block m_1 strikes to the ground at $t = 2$ s from its initial height $h = 0.5$ m. The whole system can be used to measure the gravitational acceleration around it. Using the data evaluate the gravitational acceleration, g.

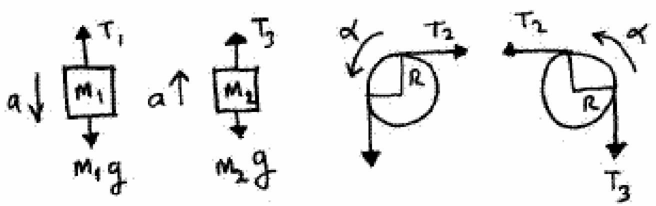


(Moment of inertia of a pulley about its CM is $I = mR^2/2$)

Acceleration of the system can be found from:

$$h = \frac{1}{2} a t^2 \rightarrow a = \frac{2h}{t^2} = \frac{2 \times 0.5}{2^2} = 0.25 \ m/s^2$$

Method I: Newton's 2nd law



for blocks: $[F = ma]$

$$m_1 g - T_1 = m_1 a \qquad (1)$$

$$T_3 - m_2 g = m_2 a \qquad (2)$$

for pulleys: $[\tau = I\alpha]$

$$(T_1 - T_2) R = I\alpha = \left(\frac{mR^2}{2}\right)\left(\frac{a}{R}\right) \qquad (3)$$

$$(T_2 - T_3) R = I\alpha = \left(\frac{mR^2}{2}\right)\left(\frac{a}{R}\right) \qquad (4)$$

Solving g:

$$g = \left(\frac{m_1 + m_2 + m}{m_1 - m_2}\right) a$$

$$= \left(\frac{18 + 17 + 5}{18 - 17}\right) 0.25$$

$$= 10 \ m/s^2$$

Method II: Conservation of energy

at $t = 0$:

$$E_i = m_1 g h$$

at $t = 2$:

$$E_f = m_2 g h + \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 + 2 \times \frac{1}{2} I w^2$$

$$E_i = E_f$$

$$m_1 g h = m_2 g h + \frac{1}{2} m_2 v^2 + \frac{1}{2} m_1 v^2 + 2 \times \left(\frac{mR^2}{2}\right) \frac{v^2}{R^2}$$

Solving for v^2:

$$v^2 = \left(\frac{m_1 - m_2}{m_1 + m_2 + m}\right) 2 g h \qquad (5)$$

On the other hand

$$v^2 = 2 a h \qquad (6)$$

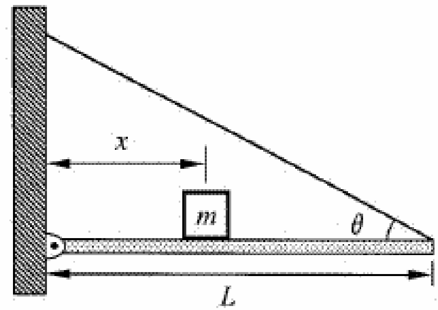
$$\therefore \left(\frac{m_1 - m_2}{m_1 + m_2 + m}\right) 2 g h = 2 a h$$

$$g = \left(\frac{m_1 + m_2 + m}{m_1 - m_2}\right) a = 10 \ m/s^2$$

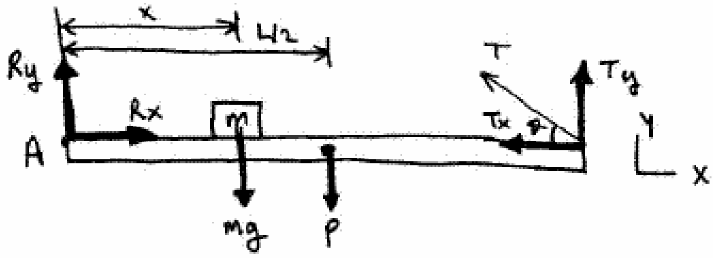
$$\boxed{g = 10 \ m/s^2}$$

QUESTION 6 (20)

One end of a uniform rod weighing 100 N and $L = 4$ m long is attached to a wall with a hinge. A wire that will break under the tension exceeding 1000 N is connected to the other end of the rod making angle $\theta = 60°$. The system is supported by this wire and a block of mass $m = 100$ kg is placed on the rod as shown in the Figure.



(a) Draw the free body diagram of the rod.



$P = 100$ N

$mg = (100)(9.8) = 980$ N

$T_x = T\cos\theta$

$T_y = T\sin\theta$

(b) Find the magnitude of the tension (T) in the wire and the magnitude of the reaction force (R) exerted by the wall on the rod, when the block is at $x = 2$ m.

$$\Sigma F_x = R_x - T_x = 0 \quad (1)$$

$$\Sigma F_y = T_y + R_y - mg - P = 0 \quad (2)$$

$$\Sigma \tau_A = T_y L - mg\,x - P\frac{L}{2} = 0 \quad (3)$$

From eqn. (3)

$$T_y = mg\,x/L + P/2$$

$$= (980)\,2/4 + 100/2$$

$$= 540 \text{ N}$$

$$R_x = T_x = \frac{T_y}{\tan\theta} = \frac{540}{\tan 60°} = 312 \text{ N}$$

from equ. (2)

$$R_y = mg + P - T_y$$

$$= 980 + 100 - 540$$

$$= 540 \text{ N}$$

$$\therefore T = \sqrt{T_x^2 + T_y^2}$$

$$= \sqrt{312^2 + 540^2} = 624 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{312^2 + 540^2} = 624 \text{ N}$$

$T =$ 624 N

$R =$ 624 N

(c) Determine the maximum distance (x) from the wall in which the wire will not break.

from eqn (3)

$$x = \frac{(T_y - P/2)L}{mg}$$

$$= \frac{(T\sin\theta - P/2)L}{mg}$$

$$x_{max} = \frac{(T_{max}\sin\theta - P/2)L}{mg}$$

$$= \frac{(1000\sin 60° - 100/2)\,4}{980}$$

$$= 3.33 \text{ m}$$

$x =$ 3.33 m