UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
Second Midterm Exam **FALL SEMESTER**
Date: 26/12/2006 Time: 110 min.

Ques.	Mark
1	
2	
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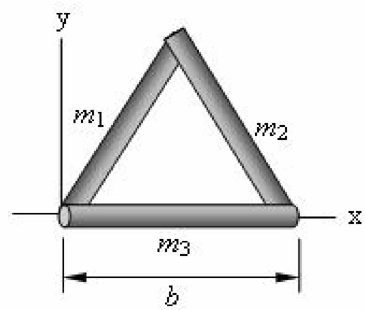
6	xxxxx
Total	
OUT OF	100

Name	Surname	Dep.	Signature
— SOLUTIONS —			

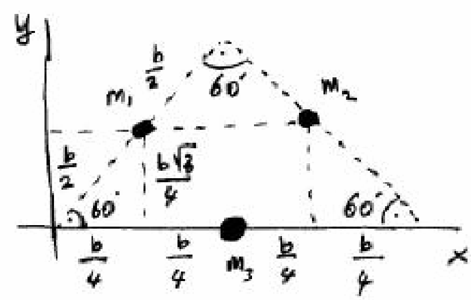
* The steps of solution of each problem should be shown clearly in the space given.
* Write your answers in boxes provided, otherwise your answer will not be considered.
* Constants: $g = 9.8\ m/s^2$; $\sin 45° = \cos 45° = 0.7$, $\sin 37° = 0.6$ $\cos 37° = 0.8$, $\cos 60° = 0.5$

QUESTION 1 (20 %)

Three uniform wires form an equilateral triangle of side $b = 20$ cm. The triangle is placed in a coordinate system as shown in Figure. Using the definition of center of mass, locate the center of mass of the triangle. Assume that the masses of the wires are given by: $m_1 = m_2 = m$ and $m_3 = 2m$.



Since the wires are uniform, the system can be assumed to be consisting of three resting particles whose properties are given in table below:



Particle	mass	x	y
m_1	m	$b/4$	$b\sqrt{3}/4$
m_2	m	$3b/4$	$b\sqrt{3}/4$
m_3	$2m$	$b/2$	0

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{mb/4 + 3mb/4 + 2mb/2}{4m} = \frac{b}{2} = \frac{20}{2} = 10\ cm$$

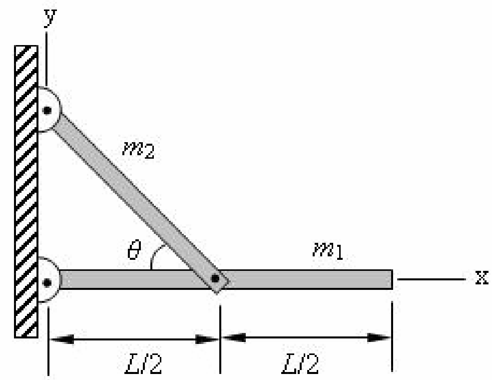
$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{mb\sqrt{3}/4 + mb\sqrt{3}/4 + 0}{4m} = \frac{b\sqrt{3}}{8} = \frac{20\sqrt{3}}{8} = 4.33\ cm$$

$x_{CM} =$ 10 cm

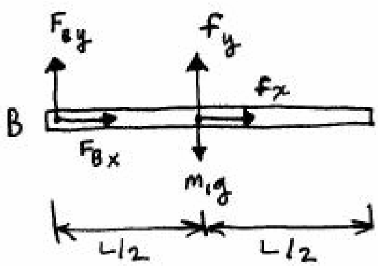
$y_{CM} =$ 4.33 cm

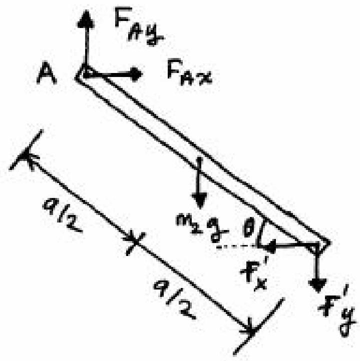
QUESTION 2 (20 %)

Two uniform beams are attached to a wall by hinges from points A and B. The beams are loosely bolted together and placed in xy plane as shown in Figure. Assume that masses of the beams are given by $m_1 = 6$ kg, $m_2 = 4$ kg and the angle between the beams is $\theta = 45^{\circ}$.



(a) Draw the free-body diagram showing all the forces acting on each beam





$$f_x = f'_x, \quad f_y = f'_y \quad \Big\} \text{ Newton's } 3^{rd} \text{ Law.}$$

$$m_1 g = (6)(9.8) = 58.8\ N$$

$$m_2 g = (4)(9.8) = 39.2\ N$$

(b) Find the magnitudes of the reaction forces exerted by the hinges on each beam at points A and B

Equilibrium conditions for m_1:

$$\Sigma F_x = F_{Bx} + f_x = 0 \qquad (1)$$

$$\Sigma F_y = F_{By} + f_y - m_1 g = 0 \qquad (2)$$

$$\Sigma \tau_B = f_y \frac{L}{2} - m_1 g \frac{L}{2} = 0 \qquad (3)$$

Equilibrium conditions for m_2:

$$\Sigma F_x = F_{Ax} - f_x = 0 \qquad (4)$$

$$\Sigma F_y = F_{Ay} - m_2 g - f_y = 0 \qquad (5)$$

$$\Sigma \tau_A = m_2 g \cos 45 \frac{a}{2} + f_x \cos 45\, a + f_y \cos 45\, a = 0 \qquad (6)$$

from (3): $f_y = m_1 g = 58.8\ N$

from (2): $F_{By} = m_1 g - f_y = m_1 g - m_1 g = 0$

from (6): $f_x = -f_y - m_2 g/2 = -58.8 - 39.2/2 = -78.4\ N$

from (5): $F_{Ay} = m_2 g + f_y = 39.2 + 58.8 = 98\ N$

from (4): $F_{Ax} = f_x = -78.4\ N$

from (1): $F_{Bx} = -f_x = +78.4\ N$

<u>Note</u>: Negative sign in some components indicates that direction must be opposite.

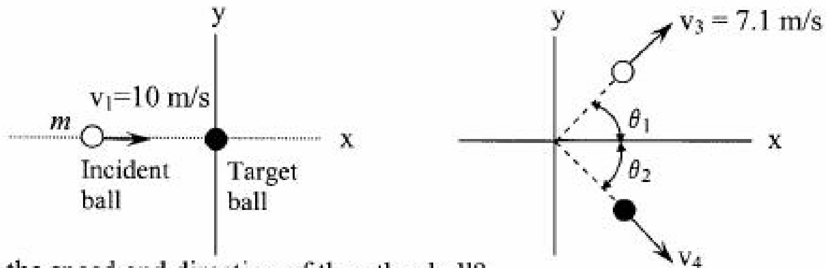
$$\therefore F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{(-78.4)^2 + (98)^2} = 125.5\ N$$

$$F_B = \sqrt{F_{Bx}^2 + F_{By}^2} = \sqrt{(78.4)^2 + 0} = 78.4\ N$$

$F_A =$	$125.5\ N$
$F_B =$	$78.4\ N$

QUESTION 3 (20 %)

A ball with mass m moving at a speed of v_1 = 10 m/s strikes an identical stationary ball as shown in Figure (left). After the collision, the speed and direction of the incident ball are given by v_3 = 7.1 m/s and $\theta_1 = 37^\circ$ respectively, see Figure (right). Assume that no external force acts on the system.



(a) What is the speed and direction of the other ball?

Conservation of momentum:

$$m V_1 = m V_3 \cos\theta_1 + m V_4 \cos\theta_2 \longrightarrow V_4 \cos\theta_2 = V_1 - V_3 \cos\theta_1 \qquad (1)$$

$$0 = m V_3 \sin\theta_1 - m V_4 \sin\theta_2 \longrightarrow V_4 \sin\theta_2 = V_3 \sin\theta_1 \qquad (2)$$

Dividing (2) by (1) yields:

$$\frac{V_4 \sin\theta_2}{V_4 \cos\theta_2} = \tan\theta_2 = \frac{V_3 \sin\theta_1}{V_1 - V_3\cos\theta_1} = \frac{(7.1)(\sin 37)}{10 - (7.1)(\cos 37)} = 1$$

$$\theta_2 = \tan^{-1}(1) = 45^\circ$$

using (2): $V_4 = \dfrac{V_3 \sin\theta_1}{\sin\theta_2} = \dfrac{(7.1)(\sin 37)}{\sin 45} = 6 \text{ m/s}$

$\boxed{v_4 = 6 \text{ m/s}}$

$\boxed{\theta_2 = 45^\circ}$

(b) Compute the total fractional kinetic energy loss in the collision

$$f = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2} m V_3^2 + \frac{1}{2} m V_4^2 - \frac{1}{2} m V_1^2}{\frac{1}{2} m V_1^2} = \frac{V_3^2 + V_4^2 - V_1^2}{V_1^2}$$

$$= \frac{(7.1)^2 + (6)^2 - (10)^2}{10^2} = -0.136$$

$$\equiv 13.6\ \%$$

$\boxed{f = -0.136}$

(c) In unit vector notation, calculate the velocity of center of mass of the balls after the collision

$$\vec{V}_3 = V_3 \cos\theta_1 \hat{\imath} + V_3 \sin\theta_1 \hat{\jmath} = 7.1(\cos 37\, \hat{\imath} + \sin 37\, \hat{\jmath}) = 5.68\hat{\imath} + 4.26\hat{\jmath} \text{ m/s}$$

$$\vec{V}_4 = V_4 \cos\theta_2 \hat{\imath} + V_4 \sin\theta_2 \hat{\jmath} = 6.0(\cos 45 \hat{\imath} - \sin 45 \hat{\jmath}) = 4.26\hat{\imath} - 4.26\hat{\jmath} \text{ m/s}$$

Definition of center of mass velocity:

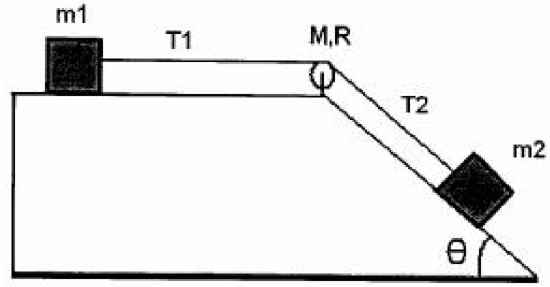
$$(m+m)\vec{V}_{CM} = m\vec{V}_3 + m\vec{V}_4 \rightarrow \vec{V}_{CM} = \vec{V}_3/2 + \vec{V}_4/2$$

$$\vec{V}_{cm} = \frac{5.68\hat{\imath} + 4.26\hat{\jmath}}{2} + \frac{4.26\hat{\imath} - 4.26\hat{\jmath}}{2} = 5\hat{\imath} \text{ m/s}$$

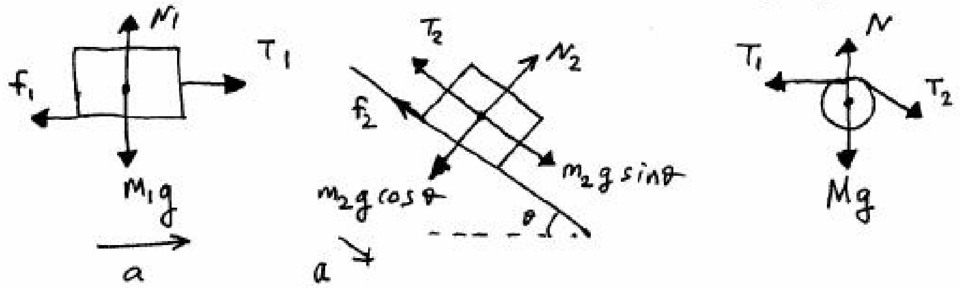
$\boxed{\vec{v}_{CM} = 5\hat{\imath} \text{ m/s}}$

QUESTION 4 (20 %)

A block of mass $m_1 = 4$ kg and a block of mass $m_2 = 12$ kg are connected by a massless string over a pulley in the shape of a disk having radius $R = 0.25$ m and mass $M = 10$ kg. These blocks are allowed to move on a fixed block–wedge of angle $\theta = 37^o$, as shown in Figure. The kinetic friction coefficient for both blocks is $\mu = 0.36$. ($I = MR^2/2$ for the pulley)



(a) Draw free-body diagrams of both blocks and of the pulley



$$f_1 = \mu m_1 g = (0.36)(4)(9.8) = 14.1 \text{ N}$$

$$f_2 = \mu m_2 g \cos\theta = (0.36)(12)(9.8)\cos 37 = 33.9 \text{ N}$$

(b) Determine the common acceleration of the two blocks

Newton's 2^{nd} Law:

$$T_1 - f_1 = m_1 a \quad (1)$$

$$m_2 g \sin\theta - f_2 - T_2 = m_2 a \quad (2)$$

$$\tau = I\alpha$$

$$(T_2 - T_1)R = \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right)$$

$$T_2 - T_1 = \frac{Ma}{2} \quad (3)$$

Adding (1), (2) and (3):

$$m_2 g \sin\theta - f_1 - f_2 = \left(m_1 + m_2 + \frac{M}{2}\right)a$$

or

$$a = \frac{m_2 g \sin\theta - f_1 - f_2}{m_1 + m_2 + M/2} = \frac{(12)(9.8)\sin 37 - 14.1 - 33.9}{4 + 12 + 10/2} = 1.07 \ m/s^2$$

$\boxed{a = 1.07 \ m/s^2}$

(c) Determine the tensions T_1 and T_2 in the string on both sides of the pulley

from (1) → $T_1 = m_1 a + f_1 = (4)(1.07) + 14.1 = 18.40 N$

from (3) → $T_2 = \frac{Ma}{2} + T_1 = \frac{(10)(1.07)}{2} + 18.4 = 23.75 \ N$

$\boxed{T_1 = 18.40 \ N}$

$\boxed{T_2 = 23.75 \ N}$

(d) Determine the angular acceleration of the pulley

$$\alpha = \frac{a}{R} = \frac{1.07}{0.25} = 4.28 \ rad/s^2$$

$\boxed{\alpha = 4.28 \ rad/s^2}$

QUESTION 5 (20 %)

A particle oscillates with simple harmonic motion according to the equation:

$$x = (6.0 \text{ m})\cos\left[(3\pi \text{ rad/s})t + \frac{\pi}{3}\text{rad}\right]$$

(a) Find the phase of the motion at time $t = 2$ s

$$\text{phase} = 3\pi t + \pi/3$$
$$= 3\pi(2) + \pi/3$$
$$= 19\pi/3 \approx 20 \text{ rad}$$

phase = 20 rad

(b) Find the displacement of the particle at $t = 2$ s

$$x(2) = 6 \cos[3\pi(2) + \pi/3] = 6 \cos[19\pi/3]$$
$$= 6 \cos[\pi/3]$$
$$= 3 \text{ m}$$

$x =$ 3 m

(c) Find the velocity at of the particle at $t = 2$ s

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{6 \cos[3\pi t + \pi/3]\right\} = -18\pi \sin[3\pi t + \pi/3]$$

$$v(2) = -18\pi \sin[19\pi/3] = -49 \text{ m/s}$$

v = − 49 m/s

(d) Find the acceleration at of the particle at $t = 2$ s

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{-18\pi \sin[3\pi t + \pi/3]\right\} = -54\pi^2 \cos[3\pi t + \pi/3]$$

$$a(2) = -54\pi^2 \cos[19\pi/3] = -266.5 \text{ m/s}^2$$

$a =$ − 266.5 m/s^2

(e) Find the frequency of the motion

$$x(t) = A\cos(\omega t + \phi)$$
$$x(t) = 6\cos(3\pi t + \pi/3)$$

Comparing these equations:

$A = 6$ m

$\omega = 3\pi$ rad/s

$\phi = \pi/3$ rad

$$f = \frac{\omega}{2\pi} = \frac{3\pi}{2\pi} = 1.5 \text{ s}^{-1}$$

f = 1.5 Hz

(f) Find the period of the motion

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = 0.67 \text{ s}$$

$T =$ 0.67 s