UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
First Midterm Exam SUMMER SCHOLL

Date: 17/07/2006 Time: 100 min.

Name	Surname	Dep.	Signature
Solutions	!		———

Ques.	Mark
1	
2	
3	
4	
5	xxxxx
6	xxxxx
Total	
Out of	100 %

- The steps of solution of each problem should be shown clearly in the space provided.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Useful constants: $g = 9.8\ m/s^2$; $\sin 37° = \cos 53° = 0.6$, $\cos 37° = \sin 53° = 0.8$

QUESTION 1 (25 %)

Given $\vec{A} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\vec{B} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

(a) Find the angle (θ) between the vectors using the definition of dot product of two vectors.

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(3)(2) + (-2)(1) + (1)(-4)}{\sqrt{3^2+2^2+1^2}\ \sqrt{2^2+1^2+4^2}} = 0$$

$$\theta = \cos^{-1}(0) = \pi/2 = 90°$$

$\boxed{\theta = 90°}$

(b) Find the angles between the vector $\vec{A}$ and positive x-, y-, and z-axis.

The magnitude of vector $\vec{A}$: $A = \sqrt{3^2+2^2+1^2} = \sqrt{14}$

$$\cos\theta_x = \frac{\vec{A}\cdot\hat{i}}{A} = \frac{(3\hat{i}-2\hat{j}+\hat{k})\cdot\hat{i}}{\sqrt{14}} = \frac{3}{\sqrt{14}} \rightarrow \theta_x = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right) = 37°$$

$\boxed{\theta_x = 37°}$

$$\cos\theta_y = \frac{\vec{A}\cdot\hat{j}}{A} = \frac{(3\hat{i}-2\hat{j}+\hat{k})\cdot\hat{j}}{\sqrt{14}} = \frac{-2}{\sqrt{14}} \rightarrow \theta_y = \cos^{-1}\left(\frac{-2}{\sqrt{14}}\right) = 122°$$

$\boxed{\theta_y = 122°}$

$$\cos\theta_z = \frac{\vec{A}\cdot\hat{k}}{A} = \frac{(3\hat{i}-2\hat{j}+\hat{k})\cdot\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}} \rightarrow \theta_z = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) = 74.5°$$

$\boxed{\theta_z = 74.5°}$

(c) Find a third vector, $\vec{C}$, that is perpendicular to the plane formed by the vectors $\vec{A}$ and $\vec{B}$.

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 2 & 1 & -4 \end{vmatrix} = 7\hat{i} + 14\hat{j} + 7\hat{k}$$

$\boxed{\vec{C} = 7\hat{i} + 14\hat{j} + 7\hat{k}}$

(d) Find the area of parallelogram formed by the vectors $\vec{A}$ and $\vec{B}$.

$$Area \equiv |\vec{C}| = \sqrt{7^2 + 14^2 + 7^2} = \sqrt{294} \approx 17\ unit^2$$

OR

$$Area \equiv |\vec{A}\times\vec{B}| = AB\sin 90° = \sqrt{3^2+2^2+1^2}\cdot\sqrt{2^2+1^2+4^2} = \sqrt{294}$$

$\boxed{Area = 17\ unit^2}$

QUESTION 2 (25 %)

Time, t(s)	Position, x(m)
0.0	1.00
0.5	4.25
1.0	6.50
1.5	7.75
2.0	8.00
2.5	7.25
3.0	5.50
3.5	2.75
4.0	-1.00

Table gives data on the position (measured in meter) of a particle moving along a straight line at various times (measured in seconds).

Assume that the particle acceleration is constant.

(a) Find an expression as for the particle position as a function of time.

General equation: $x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$; we need to find x_0, v_0 and a.

at $t = 0$: $x(0) = x_0 + 0 \cdot v_0 + \frac{1}{2} a \cdot 0^2 = 1.0 \rightarrow x_0 = 1.0$

at $t = 1$: $x(1) = 1 + v_0 + \frac{1}{2} a \rightarrow 1 + v_0 + a/2 = 6.5$ (1)

at $t = 2$: $x(2) = 1 + 2v_0 + 2a \rightarrow 1 + 2v_0 + 2a = 8.0$ (2)

Solving (1) and (2) :

$v_0 = 7.5$ m/s

$a = -4$ m/s^2

$\therefore x(t) = 1 + 7.5t - 2t^2$ (meter)

$x(t) =$ $1 + 7.5t - 2t^2$ (m)

(b) What is the position of the particle at $t = 2.3$ s?

$x(2.3) = 1 + 7.5(2.3) - 2(2.3)^2$

$= 7.67$ m

$x =$ 7.67 m

(c) What is the average velocity of the particle between $t = 3.5$ s and $t = 1.0$ s?

$$v_{avr} = \frac{x(3.5) - x(1.0)}{3.5 - 1.0} = \frac{2.75 - 6.50}{2.50} = -1.5 \text{ m/s}$$

$v_{avr} =$ -1.5 m/s

(d) What is the instantaneous velocity of the particle at $t = 2.0$ s?

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(1 + 7.5t - 2t^2\right)$$

$$= 7.5 - 4t$$

$$v(2) = 7.5 - 4(2)$$

$$= -0.5 \text{ m/s}$$

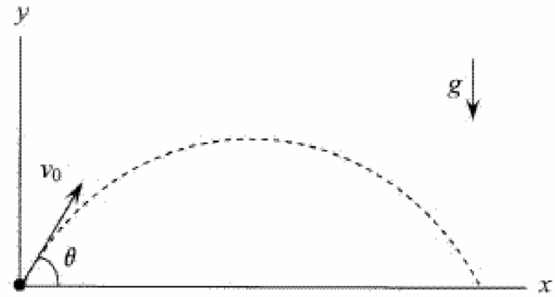
$v =$ -0.5 m/s

QUESTION 3 (25 %)

A projectile whose initial velocity is v_0 is thrown with and angle θ from the origin of the x–y plane as shown in Figure. The position vector of the projectile as a function of time is given by:

$$\vec{\mathbf{r}} = 5.6t\,\mathbf{i} + (4.2t - 4.9t^2)\,\mathbf{j} \quad \text{(meter)}$$

where t is measured in seconds.



(a) Find an expression for the velocity vector, $\vec{\mathbf{v}}$, of the projectile as a function of time, t.

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[5.6t\,\hat{i} + (4.2t - 4.9t^2)\,\hat{j}\right]$$

$$= 5.6\,\hat{i} + (4.2 - 9.8t)\,\hat{j} \quad \text{m/s}$$

$$\boxed{\vec{\mathbf{v}} = 5.6\,\hat{i} + (4.2 - 9.8t)\,\hat{j} \ \text{m/s}}$$

(b) Find an expression for the acceleration vector, $\vec{\mathbf{a}}$, of the projectile as a function of time, t.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\left[5.6\,\hat{i} + (4.2 - 9.8t)\,\hat{j}\right]$$

$$= -9.8\,\hat{j} \quad \text{m/s}^2$$

$$\boxed{\vec{\mathbf{a}} = -9.8\,\hat{j} = -g\,\hat{j} \ \text{m/s}^2}$$

(c) Calculate the initial velocity (v_0) and thrown angle (θ) of the projectile.

First way:

Velocity vector:

$$\vec{V} = v_0\cos\theta\,\hat{i} + (v_0\sin\theta - g\,t)\,\hat{j}$$

$$\vec{V} = 5.6\,\hat{i} + (4.2 - 9.8t)\,\hat{j}$$

Comparing these two equations:

$$v_0\cos\theta = 5.6 \ \text{m/s} \quad (1)$$

$$v_0\sin\theta = 4.2 \ \text{m/s} \quad (2)$$

Dividing (2) by (1):

$$\frac{v_0\sin\theta}{v_0\cos\theta} = \tan\theta = \frac{4.2}{5.6} = 0.75$$

$$\theta = \tan^{-1}(0.75) = 37^\circ$$

from (1):

$$v_0 = \frac{5.6}{\cos 37^\circ} = 7 \ \text{m/s}$$

Second way:

velocity vector at $t = 0$

$$\vec{V}_0 = \vec{V}(0) = 5.6\,\hat{i} + 4.2\,\hat{j} \ \text{m/s}$$

$$|\vec{V}_0| = v_0 = \sqrt{5.6^2 + 4.2^2} = 7 \ \text{m/s}$$

$$\tan\theta = \frac{V_{y0}}{V_{x0}} = \frac{4.2}{5.6} = 0.75$$

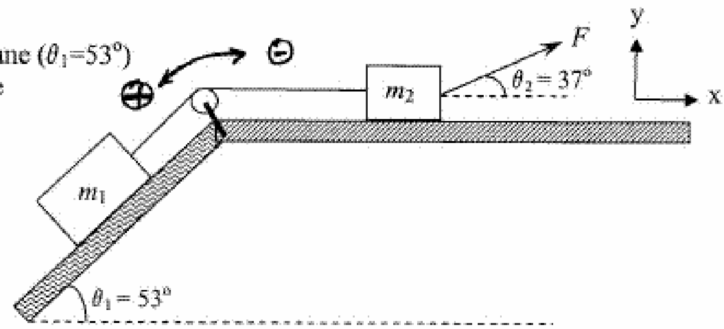
$$\theta = \tan^{-1}(0.75) = 37^\circ$$

$$\boxed{v_0 = 7 \ \text{m/s}}$$

$$\boxed{\theta = 37^\circ}$$

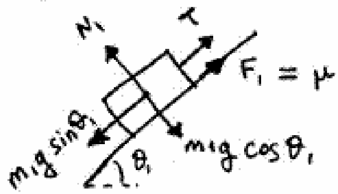
QUESTION 4 (25 %)

Two blocks $m_1 = 8$ kg on the inclined plane (θ_1=53°) and $m_2 = 5$ kg on the horizontal plane are connected by a massless cord over a frictionless pulley. A constant force F =50 N is acting on m_2 in a direction making $\theta_2 = 37°$ with the horizontal axis as shown. The coefficients of the kinetic friction between both blocks and surfaces are same and equal to 0.2.

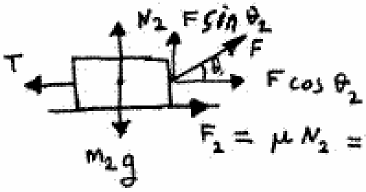


(a) Draw the free-body diagram of both blocks.

Assume that the system tends to move in the direction of ⊕.



$F_1 = \mu N_1 = \mu m_1 g \cos\theta$



$F_2 = \mu N_2 = \mu(m_2 g - F\sin\theta_2)$

(b) Find the acceleration of the block m_2.

- Newton's 2^{nd} Law for block m_1:

$$m_1 g \sin\theta_1 - F_1 - T = m_1 a$$

$$m_1 g \sin\theta_1 - \mu m_1 g \cos\theta_1 - T = m_1 a$$

$$(8)(9.8)\sin 53° - (0.2)(8)(9.8)\cos 53° - T = 8a$$

$$53.312 - T = 8a \quad (1)$$

- and for block m_2:

$$T - F_2 - F\cos\theta_2 = m_2 a$$

$$T - \mu\left[m_2 g - F\sin\theta_2\right] - F\cos\theta_2 = m_2 a$$

$$T - 0.2\left[(5)(9.8) - 50\sin 37\right] - 50\cos 37° = 5a$$

$$T - 43.8 = 5a \quad (2)$$

Add eqn. (1) by (2) to eliminate T:

$$53.312 - 43.8 = 13a$$

$$\therefore a = \frac{9.512}{13}$$

$$= 0.73 \ m/s^2$$

$a > 0$ means that our assumption in (a) is true.

$a =$ 0.73 m/s^2

(c) What is the direction of motion of the block m_2?

In the direction of $-X$