UNIVERSITY OF GAZIANTEP
DEPARTMENT OF ENGINEERING PHYSICS

EP 105 General Physics I
First Midterm Exam FALL SEMESTER
Date: 14/11/2006 Time: 110 min.

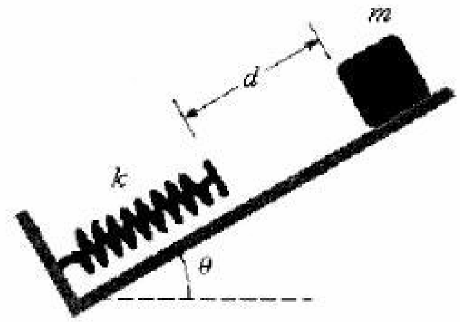
Name	Surname	Dep.	Signature
Solutions!			

Ques.	Mark
1	
2	
3	
4	
5	
6	xxxxx
Total	
OUT OF	100

- The steps of solution of each problem should be shown clearly in the space given.
- Write your answers in boxes provided, otherwise your answer will not be considered.
- Constants: $g = 9.8\ m/s^2$; $\sin 30^\circ = \cos 60^\circ = 0.5$, $\cos 30^\circ = \sin 60^\circ = 0.866$

QUESTION 1 (20 %)

A mass $m = 3$ kg starts from rest and slides a distance $d = 35$ cm down a frictionless incline of angle $\theta = 30^\circ$. While sliding, it contacts an unstressed spring of negligible mass, as shown in Figure. The mass slides an additional distance x as it is brought momentarily to rest by compression of the spring of force constant $k = 400$ N/m.



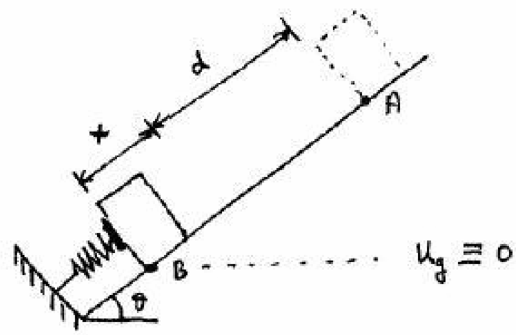
Using the conservation of mechanical energy, determine how much the spring is compressed.

$$E_A = E_B$$

$$\underbrace{K_A}_{0} + U_A = \underbrace{K_B}_{0} + U_B$$

$$U_A = U_B$$

$$mg\,[d + x]\sin\theta = \frac{1}{2} k x^2$$



$$(3)(9.8)\,[0.35 + x]\sin 30 = \frac{1}{2}\, 400\, x^2$$

$$14.7\,[0.35 + x] = 200\, x^2$$

OR

$$13.6\, x^2 - x - 0.35 = 0$$

Solutions of the quadratic equation:

$$x_{12} = \frac{+1 \mp \sqrt{1^2 - 4\,(13.6)(-0.35)}}{(2)\,(13.6)} = \frac{1 \mp \sqrt{20}}{27.2} < \begin{matrix} x_1 = -0.13\ m & ✗ \\ x_2 = +0.20\ m & ✓ \end{matrix}$$

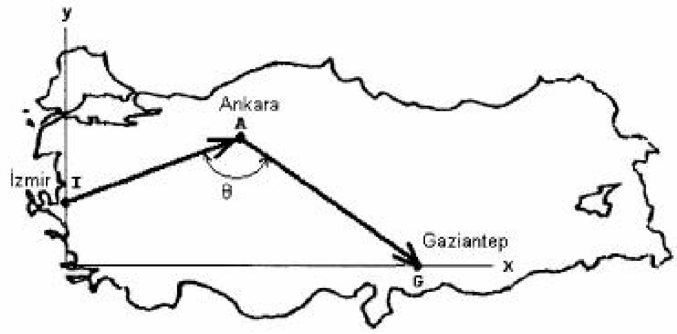
Negative root is unphysical for our configuration!
Hence, discard the negative solution.

$\boxed{x = \quad 0.2\ m}$

QUESTION 2 (20 %)

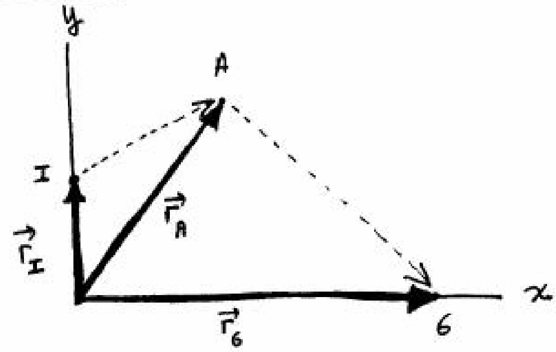
An airplane begins a trip from İzmir (point I) by first traveling to Ankara (point A) and then traveling to Gaziantep (point G). The trips can be indicated by the vectors $\vec{IA}$ and $\vec{AG}$ on the xy plane as shown in the figure. The coordinates of the cities on the xy plane are given by

I(0, 230 km),
A(400 km, 480 km) and
G(1000 km, 0) respectively.



(a) Express the displacement vectors $\vec{IA}$ and $\vec{AG}$ in unit vector notation.

$$\vec{IA} = \vec{r}_A - \vec{r}_I = (400\hat{i} + 480\hat{j}) - (0\hat{i} + 230\hat{j})$$
$$= 400\hat{i} + 250\hat{j} \ \text{(km)}$$



$$\vec{AG} = \vec{r}_G - \vec{r}_A = (1000\hat{i} + 0\hat{j}) - (400\hat{i} + 480\hat{j})$$
$$= 600\hat{i} - 480\hat{j} \ \text{(km)}$$

$\boxed{\vec{IA} = 400\hat{i} + 250\hat{j} \ \text{(km)}}$

$\boxed{\vec{AG} = 600\hat{i} - 480\hat{j} \ \text{(km)}}$

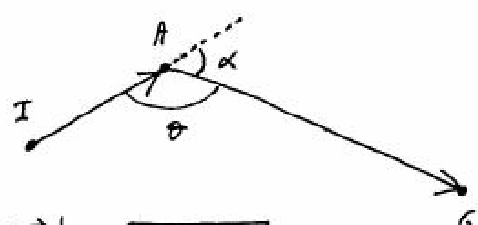
(b) Find the vector (in unit vector notation) defining the displacement from İzmir to Gaziantep (i.e. find $\vec{IG}$)

$$\vec{IG} = \vec{IA} + \vec{AG} = (400\hat{i} + 250\hat{j}) + (600\hat{i} - 480\hat{j})$$
$$= 1000\hat{i} - 230\hat{j} \ \text{(km)}$$

$\boxed{\vec{IG} = 1000\hat{i} - 230\hat{j} \ \text{(km)}}$

(c) Evaluate the angle θ in the Figure by using the definition of dot product of the vectors $\vec{IA}$ and $\vec{AG}$.

$$\cos\alpha = \frac{\vec{IA} \cdot \vec{AG}}{|\vec{IA}||\vec{AG}|}$$
$$= \frac{(400\hat{i} + 250\hat{j}) \cdot (600\hat{i} - 480\hat{j})}{(471)\ (768)}$$
$$= \frac{400 \times 600 - 250 \times 480}{361728}$$
$$= 0.332$$



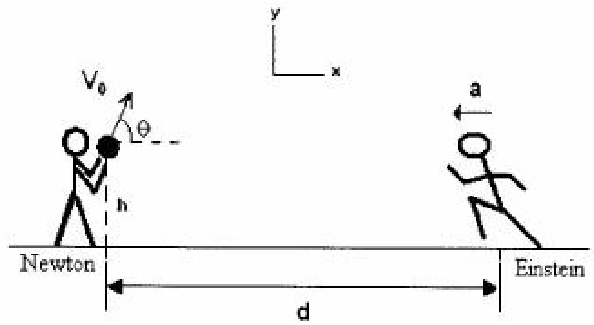
$$|\vec{IA}| = \sqrt{400^2 + 250^2} = 471 \text{ km}$$
$$|\vec{AG}| = \sqrt{600^2 + 480^2} = 768 \text{ km}$$

$$\alpha = \cos^{-1}(0.332) \approx 70^\circ$$
$$\theta = 180 - 70 = 110^\circ$$

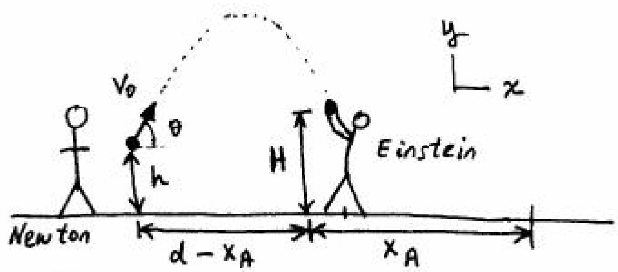
$\boxed{\theta = 110^\circ}$

QUESTION 3 (20 %)

Newton has thrown a ball upward, at a horizontal angle θ, from a height of h = 1.6 m above the ground. At the moment the ball was thrown Newton told Einstein to catch it and Einstein has accelerated from rest at a = 3 m/s^2 directly toward the ball, at a distance of d = 20 m from Newton. Einstein has started running and caught the ball at a height of 2 m above the ground 2 s later.



(a) What distance does Einstein run to catch the ball?



$$x_A = \underbrace{x_{A0}}_{0} + \underbrace{v_{A0}}_{0} t + \frac{1}{2} a t^2$$
$$= \frac{1}{2} a t^2$$
$$= \frac{1}{2}(3)(2)^2$$
$$= 6\ m$$

$x_A =$ 6 m

(b) At what angle was the ball thrown?

Equations of motion for the ball : $\left.\begin{aligned} d - x_A &= v_0 \cos\theta\, t \\ H &= h + v_0 \sin\theta\, t - \frac{1}{2} g t^2 \end{aligned}\right\}$ $\quad v_0 \sin\theta\, t = H - h + \frac{g t^2}{2} \quad (1)$

$v_0 \cos\theta\, t = d - x_A \quad (2)$

Dividing eqn (1) by (2) yields:

$$\tan\theta = \frac{H - h + g t^2/2}{d - x_A} = \frac{2 - 1.6 + 9.8(2)^2/2}{20 - 6} = \frac{10}{7}$$

$$\therefore\ \theta = \tan^{-1}(10/7)$$
$$= 55^\circ$$

$\theta =$ 55°

(c) At what initial speed was the ball thrown?

From equation (2)

$$v_0 = \frac{d - x_A}{t \cos\theta} = \frac{20 - 6}{2 \cos 55} = 12.2\ m/s$$

$V_0 =$ 12.2 m/s

(d) What was the highest point, above the ground, that the ball reached?

y-component of the velocity of the ball:

$$= v_0 \sin\theta - g t$$

At $t = t_{max}$ $\quad v_y = 0$

$$0 = v_0 \sin\theta - g t_{max}$$

$$t_{max} = \frac{v_0 \sin\theta}{g} = \frac{(12.2)\sin 55}{9.8}$$
$$= 1\ s$$

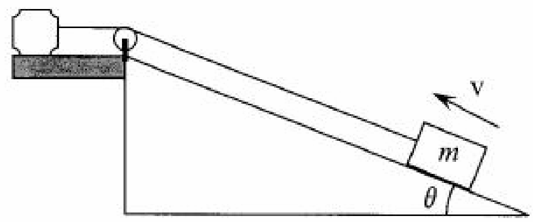
Therefore corresponding y-position is:

$$H_{max} = h + v_0 \sin\theta\, t_{max} - \frac{1}{2} g t_{max}^2$$
$$= 1.6 + (12.2)(\sin 55)(1) - \frac{1}{2} 9.8 (1)^2$$
$$= 6.7\ m$$

$H_{max} =$ 6.7 m

QUESTION 4 (20 %)

A 150 kg block is pulled up an $\theta = 30^\circ$ inclined plane at constant speed of v = 3 m/s winch as in Figure. The block comes to rest after traveling 5 m along the plane and the coefficient of kinetic friction between the block and plane is $\mu = 0.20$.

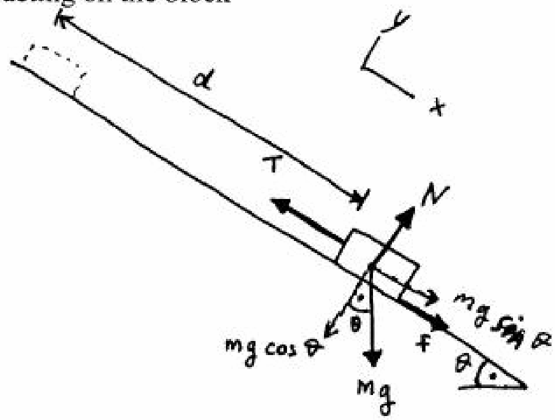


(a) Draw the free body diagram showing all the forces acting on the block

$$f = \mu N$$

$$mg = (150)(9.8) = 1470\ N$$

$$d = 5\ m$$



(b) Calculate the work done by the winch

Newton's 2nd law for the block:

$$\Sigma F_x = mg\sin\theta + f - T = 0$$

$$\Sigma F_y = N - mg\cos\theta = 0$$

Solving for T:

$$T = mg\left[\sin\theta + \mu\cos\theta\right]$$

$$= 1470\left[\sin 30 + 0.2\cos 30\right]$$

$$= 990\ N$$

Work done by T:

$$W_w = T d \cos 0^\circ$$

$$= (990)(5)(1)$$

$$= 4950\ J$$

$$\boxed{W_w = 4950\ J}$$

(c) Calculate the work done by the frictional force

$$W_f = f d \cos 180^\circ = -\mu m g d \cos\theta$$

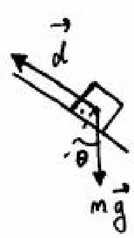
$$= -(0.2)(1470)(5)\cos 30$$

$$= -1275\ J$$

$$\boxed{W_f = -1275\ J}$$

(d) Calculate the work done by the weight of the block

method I



$$W_g = m\vec{g} \cdot \vec{d}$$

$$= mgd\cos(90+\theta)$$

$$= -mgd\sin\theta$$

$$= -(1470)(5)\sin 30$$

$$= -3675\ J$$

method II

$$W_g = (mg)_x\ d\ \cos 180^\circ$$

$$= (mg\sin\theta)\ d\ (-1)$$

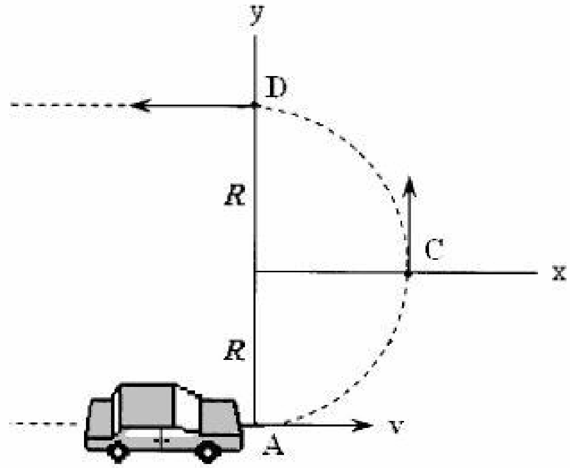
$$= -mgd\sin\theta$$

$$= -3675\ J$$

$$\boxed{W_g = -3675\ J}$$

QUESTION 5 (20 %)

A car initially traveling eastward makes a U-turn by traveling in a semi-circular path ACD at uniform speed as shown in the Figure. The radius of the curvature is $R = 35$ m, and the car completes the U-turn in 10 s.

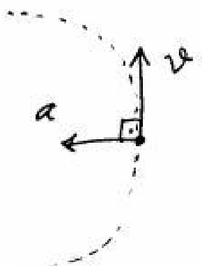


(a) What is the speed of the car?

$$v = \frac{|\widehat{ACD}|}{t} = \frac{\pi R}{t} = \frac{(\pi)(35)}{10} = 11 \text{ m/s}$$

$\boxed{v = 11 \text{ m/s}}$

(b) What is the magnitude and direction of the car's acceleration when it is at point C?



$$a = \frac{v^2}{R} = \frac{(11)^2}{35} = 3.5 \text{ m/s}^2$$

in the direction of westward or $-x$

$\boxed{a = 3.5 \text{ m/s}^2}$

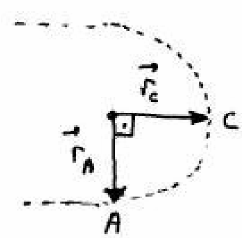
$\boxed{\text{Direction: } -X}$

(c) What is the magnitude of the car's average velocity between points A and C?

$$\vec{V}_{AVR} = \frac{\vec{r}_C - \vec{r}_A}{t/2} = \frac{R\hat{i} - (-R\hat{j})}{t/2} = \frac{2R}{t}(\hat{i}+\hat{j})$$

$$\vec{V}_{AVR} = \frac{(2)(35)}{10}(\hat{i}+\hat{j}) = 7(\hat{i}+\hat{j}) \text{ m/s}$$

$$|\vec{V}_{AVR}| = \sqrt{7^2+7^2} = 9.9 \text{ m/s}$$



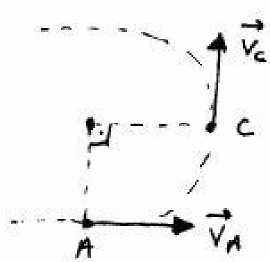
$\boxed{v_{AVR} = 9.9 \text{ m/s}}$

(d) What is the magnitude of the car's average acceleration between points A and C?

$$\vec{a}_{AVR} = \frac{\vec{V}_C - \vec{V}_A}{t/2} = \frac{v\hat{j} - v\hat{i}}{t/2} = \frac{2v}{t}(\hat{j}-\hat{i})$$

$$\vec{a}_{AVR} = \frac{(2)(11)}{10}(\hat{j}-\hat{i}) = 2.2(\hat{j}-\hat{i}) \text{ m/s}^2$$

$$|\vec{a}_{AVR}| = \sqrt{(2.2)^2+(2.2)^2} = 3.1 \text{ m/s}^2$$



$\boxed{a_{AVR} = 3.1 \text{ m/s}^2}$